



# Portfolio Construction

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## Robust Game Theory

June 1, 2006

Classical game theory, which is useful in understanding market behavior, relies on what have been described as four “essential elements” — who is playing, how much information they possess, the actions available to them, and the payoffs for each outcome.

In this paper, we propose a model of games in which players have incomplete information, and we offer ideas on how to optimize outcomes. We address incomplete-information games without private information as well as those involving potentially private information.

Our robust optimization model relaxes the assumptions of Harsanyi’s Bayesian games model and gives a notion of equilibrium that subsumes the ex post equilibrium concept. We also prove the existence of equilibria in any such robust finite game, when the payoff uncertainty set is bounded.

For any robust finite game with bounded polyhedral payoff uncertainty set and finite type spaces, we formulate the set of equilibria as the dimension-reducing, component-wise projection of the solution set of a system of multilinear equations and inequalities. We suggest a computational method for approximately solving such systems and give numerical results of the implementation of this method.

Furthermore, we describe a special class of robust finite games, whose equilibria are precisely those of a related complete-information game with the same number of players and the same action spaces. Using illustrative examples of robust games from this special class, we compare properties of robust finite games with those of their Bayesian-game counterparts. Moreover, we prove that symmetric equilibria exist in symmetric, robust finite games with bounded uncertainty sets.

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