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Still Not Cheap: *Portfolio Protection in Calm Markets*

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Still Not Cheap: *Portfolio Protection in Calm Markets*

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Recent S&P 500 Index volatility is near all-time lows. So, too, is the VIX Index, the Chicago Board Options Exchange's measure of market volatility. Options therefore appear cheap. Many market commentators, especially in financial media, present these observations and conclude that there is now a rare opportunity to cheaply buy put options for protection.

Exhibits 1 and 2 plot, respectively, the S&P 500 Index's 21-business-day annualized realized volatility and the VIX Index.[1] On June 30, 2014, the S&P 500's 21-day realized volatility was a paltry 5.8% annualized. Its volatility has been higher than this level 96% of the time going back to 1950. Similarly, the VIX Index was 11.6% on the same date, which is lower than 95% of its history going back to 1990.

Buying a put option provides portfolio insurance, but usually at a significant price. Calm environments present an opportunity to purchase put options at relatively lower prices. Buying put options also provides long volatility exposure when risk may increase due to mean-reverting tendencies.[2] Presented in this light, buying put options certainly appears compelling.

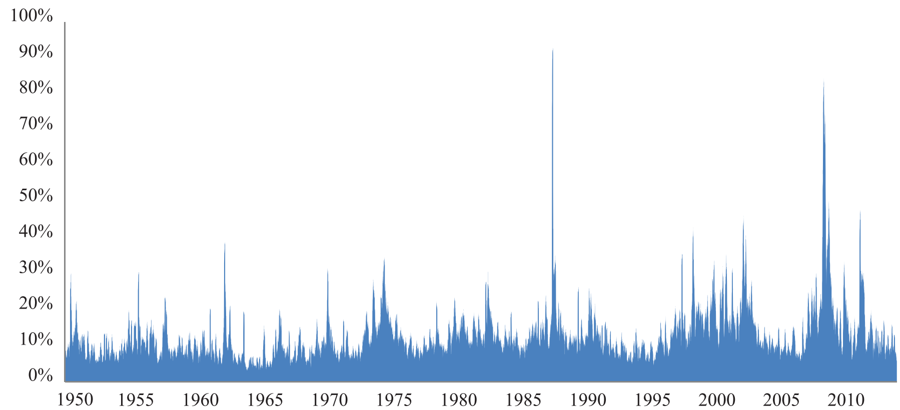
However, we cannot consider price in a vacuum. If we were to tell you that a stock's price of $11.60 is near its all-time low, would you be able to conclude whether it is a good buy? Not without some indication of its true fundamental value, such as book value, earnings, or dividends. An option is no different. Its price absent the context of its fundamental value is nearly useless information. An option's fundamental value is directly tied to its underlying security's realized volatility through the option's expiration. An S&P 500 Index option priced at 11.6% implied volatility when the index realizes 5.8% volatility is in fact very costly.

It doesn't matter if implied volatility is at or near its historical low. It doesn't matter if realized volatility is expected to increase. It doesn't even matter if realized volatility actually does increase over the option's life. What does matter is the option's purchase price (implied volatility) relative to its fundamental value (ex post realized volatility). In fact, this spread is so closely tied to the compensation paid to the option seller by the purchaser that it is widely referred to as the volatility risk premium.

This does not necessarily imply that buying an option is irrational. An investor may have constraints or preferences that justify option purchases. However, in order to appropriately evaluate the cost/benefit tradeoff, investors must first understand how purchasing put options may impact their portfolios' expected returns.

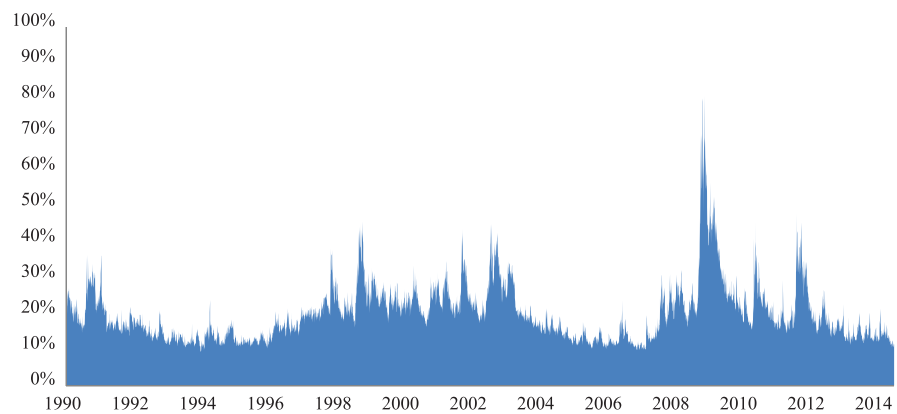
This article empirically investigates the claim that put options are cheap when equity

Exhibit 1
S&P 500 Index 21-Day Annualized Volatility, January 3, 1950, to July 31, 2014



Sources: AQR, Standard and Poor's.

Exhibit 2
CBOE Volatility Index (VIX), January 2, 1990, to July 31, 2014



Sources: AQR, Chicago Board Options Exchange.

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volatility and their options' prices are low. We begin by verifying that put options have historically been unconditionally expensive and report full sample results for context. We then investigate the magnitude of the volatility risk premium in different volatility regimes. Next, we analyze one implementation of a protective put strategy, which buys 5% out-of-the-money monthly put options and holds them until they expire. We begin by analyzing the volatility risk premium and options purchased on the S&P 500 Index. We then extend our analysis to include nine additional global equity indexes. Thus far, our analysis is focused on the cost side of owning put options. We conclude our article by considering the benefit side of the equation. We consider the impact of black swan events on protective put options. Specifically, we determine how often an October 1987 magnitude market crash has to occur, starting from actual history as the baseline case, for protective put options to break even.

Volatility Risk Premium

The most common ex post value measure for put options is the volatility risk premium, which is option implied volatility, as indicated by the VIX Index level minus the S&P 500's realized volatility over the coincident period. Exhibit 3 plots the volatility risk premium over the period January 2, 1990, through June 30, 2014, in which VIX Index data are available. Over this period, the volatility risk premium has averaged +3.4% and has been positive 88% of the time.[3] Investors who heed analysts' recommendation to purchase options are not only long volatility—they also face long odds of benefiting from the option purchase.

Gârleanu et al. [2009] show that this volatility risk premium may be due to aggregate net demand for options. Market makers cannot perfectly hedge their inventory, so they require compensation for their remaining inventory risk. As a result, aggregate end-user demand for options affects their expensiveness. If when markets are calm and the prices of options are low, investors who normally shy away from purchasing options are lulled into doing so (increased demand) and those who are normally willing to sell options are afraid of doing so (reduced supply), then the Garleanu et al. [2009] model predicts upward pressure on the volatility risk premium. Demand-based option pricing tells us that the more investors espouse the view that calm times are bad for option selling and good for option buying, the more likely they are to be wrong.

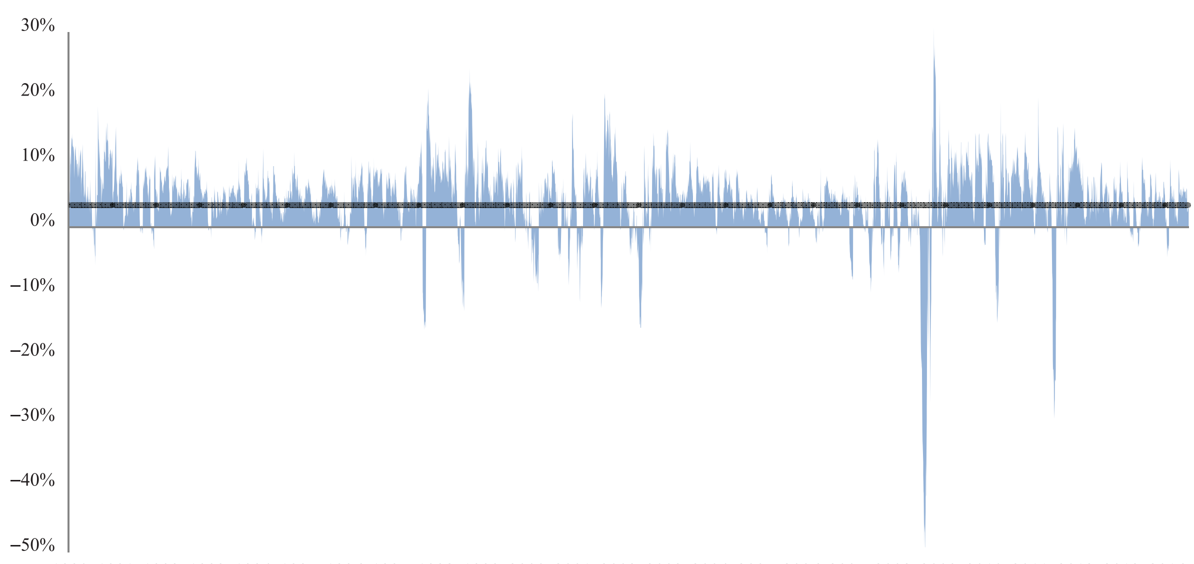
A protective put option's volatility risk premium has reduced the portfolio's risk-adjusted returns. Exhibit 4 reports summary statistics for a protective put strategy that is long the S&P 500 Index and protects the position by purchasing monthly put options that are 5% out-of-the-money. Buying a put option reduces the portfolio beta from 1.0 to 0.72. More importantly, the put option protects against instantaneous return shocks; whereas the portfolio beta to positive S&P 500 returns is 0.79, its beta to downside is 0.47.

For additional insight, we decompose the protected portfolio strategy returns into three components as suggested by Israelov and Nielsen [2015]. The first component is passive S&P 500 exposure. The second component is dynamic S&P 500 exposure due to the put option's time-varying equity exposure. These two

EXHIBIT 3

Volatility Risk Premium, January 2, 1990, to July 31, 2014

The volatility risk premium is defined as the VIX Index minus the S&P 500 Index's subsequent annualized volatility. The horizontal line corresponds to the average ex post volatility risk premium over the time period. The average volatility risk premium is computed as the square root of the average implied variance (VIX squared) minus the square root of the average coinciding realized variance.



Sources: AQR, Chicago Board Options Exchange, Standard and Poor's.

EXHIBIT 4

Summary Statistics, March 15, 1996, to June 30, 2014

The table shows summary statistics for a protective put strategy that is long the S&P 500 Index and long 5% out-of-the-money front-month S&P 500 put options, sized to unit leverage, and held to expiry. These returns are decomposed into three components: passive S&P 500 exposure, dynamic S&P 500 exposure due to the put option's time-varying equity exposure, and long volatility exposure. Volatility, beta, and skew are computed using overlapping 21-day returns.

		Decomposition of S&P 500 with Protective Put		
	S&P 500 with Protective Put 5% Out of the Money	S&P 500 Exposure (Static)	S&P 500 Exposure (Dynamic)	Long Volatility Exposure
Annualized Excess Return	2.7%	5.2%	–0.9%	–2.0%
Annualized Volatility	12.9%	14.0%	4.8%	2.4%
Sharpe Ratio	0.12	0.37	–0.18	–0.83
Beta to S&P 500 Index	0.72	0.84	–0.09	–0.03
- Upside Beta	0.79	0.84	–0.06	0.01
- Downside Beta	0.47	0.85	–0.28	–0.10
Skew	–0.1	–0.7	3.1	6.0

Sources: AQR, Option Metrics, Chicago Board Options Exchange, Standard and Poor's.

components share a common attribute. They can be attained by trading the S&P 500 Index—the first passively and the second dynamically (on a pre-determined basis)—and so can be accomplished without options. The third component is long volatility exposure, which can only be obtained via exposure to a nonlinear instrument, such as a put option.

The protected portfolio starts from 0.84 passive equity exposure, which provides 5.2% annualized excess returns. The put option's dynamic equity exposure reduces the overall beta of the strategy by 0.09, but more importantly, it reduces the downside beta of the strategy by 0.28. It reduces annual returns by a statistically insignificant 0.9%, and we do not believe there is a compelling economic reason to expect this component to lose money (on a beta-adjusted basis).

The most interesting element of the decomposition is the third component, which is specific to the option position: the long volatility exposure. Our analysis of protective put options in the remainder of the article focuses exclusively on the long volatility exposure. Although it does reduce downside beta exposure by 0.10, it does so at significant expense. The long volatility exposure reduces performance by 2.0% per year and is a negative 0.83 Sharpe ratio strategy. We believe this is a significant price to pay for a 0.10 reduction in downside beta. For context, an alternative approach could simply sell 10% of the existing equity position, reducing both upside and downside beta by 0.10; the cost to the portfolio in reduced equity risk premium is approximately 0.6% assuming a 6% annualized equity risk premium. For this reason, buying put options is often criticized as a tail protection strategy. In our opinion, such strategies are simply too expensive.

Volatility Regimes

Exhibits 3 and 4 confirm that long volatility exposure is costly on average, a result that is well established.[4] However, many investors believe the current environment is different. They suggest that option prices are low because volatility is low and volatility may revert toward its long-term average.

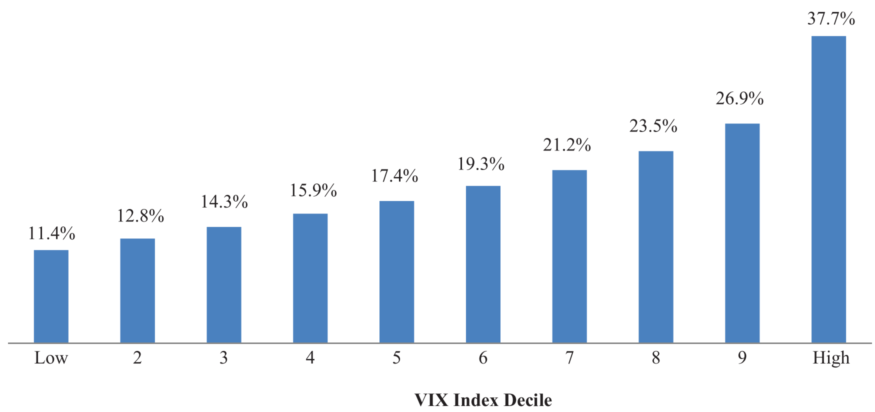
We look to the past to understand the volatility risk premium in similar risk environments. We begin by reporting, in Exhibit 5, the average VIX Index in each bucket after sorting into buckets according to the VIX Index. The June 30, 2014's VIX Index level of 11.6% resides in the lowest risk bucket.

Temporarily putting aside implementation details and the complicating path dependence of option positions, a long option position is expected to be profitable when the ex post volatility risk premium is negative, i.e., when future realized volatility has been higher than the starting level of implied volatility. We've seen that the volatility risk premium is positive 88% of the time and is significantly positive on average. Exhibit 6 plots

EXHIBIT 5

VIX Index Levels (bucketed by VIX index level), January 2, 1990, to June 30, 2014

Average VIX Index is computed after sorting the VIX Index into deciles.



Sources: AQR, Chicago Board Options Exchange.

the ex post volatility risk premium in each decile after sorting by the VIX Index at the beginning of the period (all VIX Index sorts include a one-day lag to avoid overlapping data biasing results).[5] The average volatility risk premium is positive across the board and generally increases with the VIX Index. The average volatility risk premium in the lowest three buckets is 3.1%, not that different than the 3.4% average across all buckets.

Even in the lowest VIX Index decile, the spread between the VIX Index and realized volatility is a positive 2.5%. Option prices may be lower, but they remain expensive in the sense that the long volatility component of one-month options is expected to have negative returns.

A potential counterargument may concede that options remain expensive but that their purchase is justified because they can protect against the same-sized shock at a much lower cost. Hurricane insurance in Nebraska may also provide the same protection as hurricane insurance in Florida at a much lower cost, but that doesn't necessarily make it a good value. The risks are different. Certainly we may see a significant volatility event in low-risk environments, but history tells us it is much less likely.

To aid an understanding of why, Exhibit 7 plots the distribution of the ex post volatility risk premium in each volatility bucket. The 80% confidence intervals are positively biased, consistent with the positive volatility risk premium shown in Exhibit 6. The volatility risk premium is more variable when implied volatility is high. Its 80% confidence interval is 5% wide in the lowest implied volatility decile and 19% wide in the highest decile. In the lowest-risk environment, the most extreme outcome had realized volatility 8% higher than implied volatility. In the highest risk environment, the most extreme outcome occurred when realized volatility was 49% higher than implied volatility. Although owning a put option provides the same contractual protection in each decile per se, the distribution of outcomes across volatility environments has been very different.

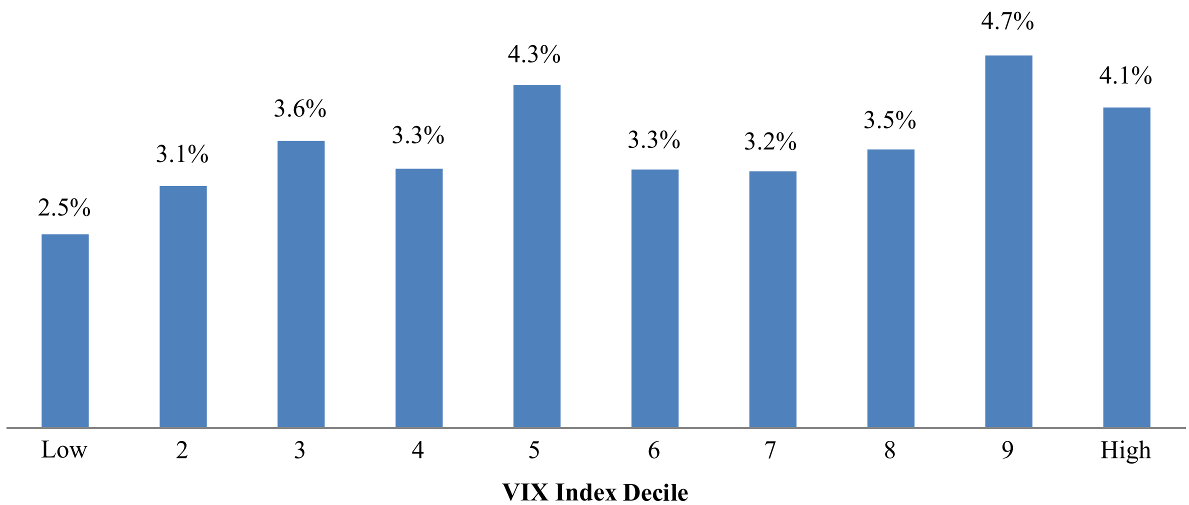
The volatility risk premium is not a strategy return, but it is related to one in the following way. When positive, it indicates that a very specific strategy implementation (long a 30-day variance swap) loses money. It also indicates that buying options and delta-hedging to remove the options' variable equity exposure is expected to lose money on average, but implementation decisions surrounding strike and maturity selection and their corresponding effect on path dependence adds basis risk.

Yet, buying put options is the most direct method portfolio managers can use to protect their portfolios. Hence, we now analyze put option performance in different volatility environments. Specifically, each month on option expiration, we buy a 5% out-of-the-money put option and the option position is delta-hedged each day so that it has no equity exposure.

Exhibit 6

Ex Post Volatility Risk Premium (bucketed by VIX index level), January 2, 1990, to June 30, 2014

Average ex post volatility risk premium in each decile after bucketing by the VIX Index at the beginning of the period (all VIX Index sorts include a one-day lag to avoid overlapping data biasing results). Volatility risk premium defined as VIX Index minus the S&P 500 Index's subsequent annualized volatility. The average volatility risk premium is computed as the square root of the average implied variance (VIX squared) minus the square root of the average coinciding realized variance.

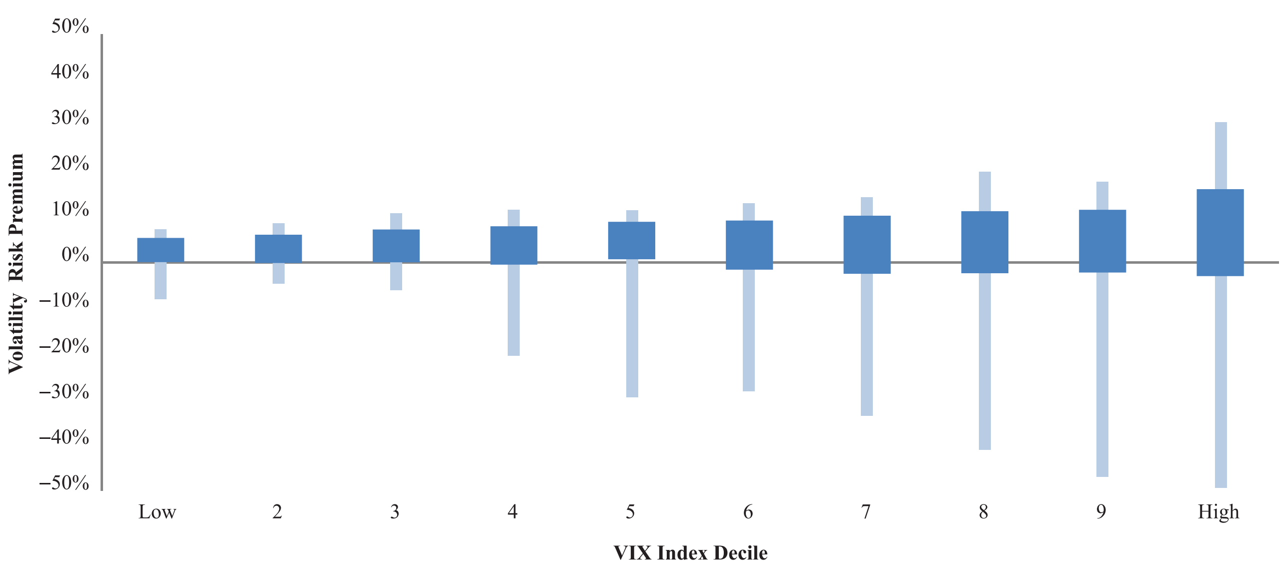


Sources: AQR, Chicago Board Options Exchange, Standard and Poor's.

Exhibit 7

Ex Post Volatility Risk Premium Distribution (bucketed by VIX index level), March 15, 1996, to June 30, 2014

The chart shows the ex post volatility risk premium distribution in each decile after bucketing by the VIX Index at the beginning of the period (all VIX Index sorts include a one-day lag to avoid overlapping data biasing results). Volatility risk premium is defined as VIX Index minus the S&P 500 Index's subsequent annualized volatility. The box depicts the 80% confidence interval, and the whiskers are the minimum and maximum values within each bucket.

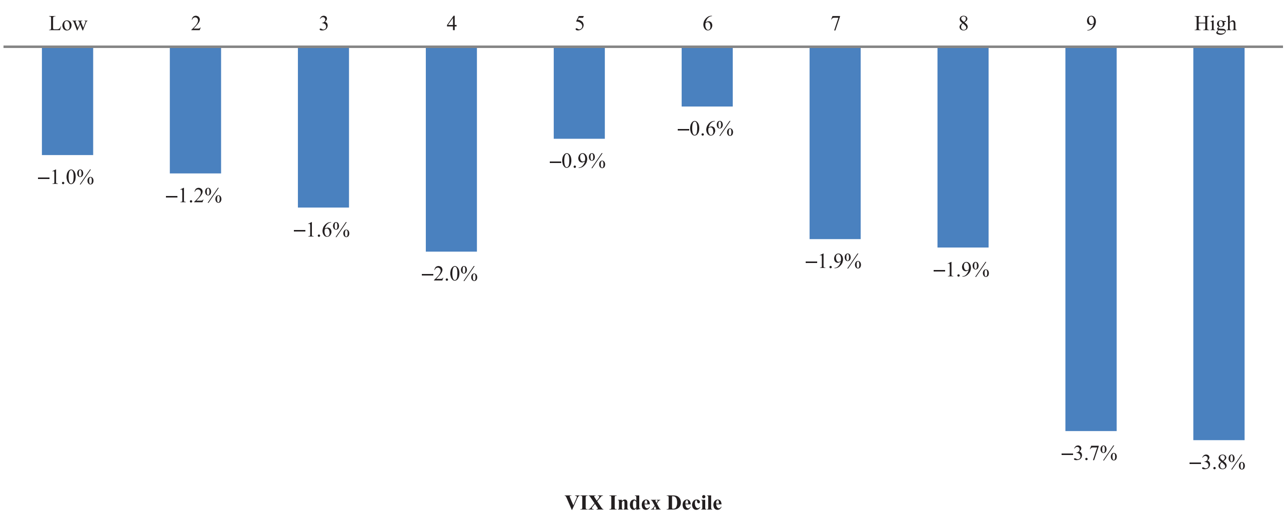


Sources: AQR, Chicago Board Options Exchange, Standard and Poor's.

EXHIBIT 8

Long One-Month Delta-Hedged 5% Out-of-the-Money Put Options Annualized Return (bucketed by VIX index), March 15, 1996, to June 30, 2014

Average annualized returns are computed after sorting the backtest returns by VIX Index into deciles. The backtest bought front-month SPX put options, selected to be 95% out of the money, sized to unit leverage, and held until expiry. The options were delta-hedged daily.

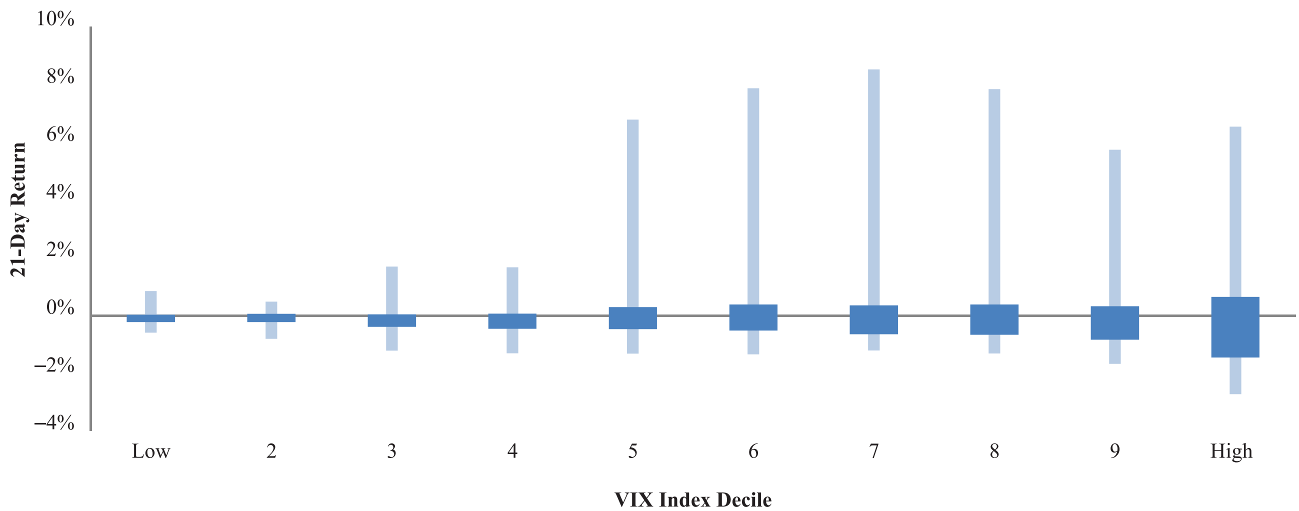


Sources: AQR, Option Metrics, Chicago Board Options Exchange, Standard and Poor's.

EXHIBIT 9

Long One-Month Delta-Hedged 5% Out-of-the-Money Put Options Return Distribution (bucketed by VIX index), March 15, 1996, to June 30, 2014

The 21-day return distributions are computed after sorting the backtest returns by VIX Index into deciles. The box depicts the 80% confidence interval, and the whiskers are the minimum and maximum values within each bucket. The backtest bought front-month SPX put options, selected to be 95% out of the money, sized to unit leverage, and held until expiry. The options were delta-hedged daily.



Sources: AQR, Option Metrics, Chicago Board Options Exchange, Standard and Poor's.

Exhibit 8 plots the delta-hedged put option's average annualized return across implied volatility deciles. Average returns are negative in each decile, and losses generally increase with the starting level of implied volatility, consistent with the findings for the volatility risk premium shown in Exhibit 6. Being long volatility by owning put options has historically cost, respectively, 1.0% and 1.2% annualized in the two lowest-risk buckets (calm markets), slightly more than half of the full sample average 1.9% annualized cost reported in Exhibit 4. Less-expensive options in calm markets do not necessarily mean that investors are getting a good deal.

Averages tell only part of the story. Put options are purchased in order to generate outsized positive returns at the right time. Exhibit 9 plots the put option's minimum and maximum returns and 80% return confidence region in each bucket after sorting on the VIX Index. Visual inspection clearly indicates that whatever protection puts have historically provided has not occurred in calm environments. In fact, the maximum 21-day return in the lowest four deciles is only 1.7%. Paying more than 1% of net asset value (NAV) per year to buy these options hardly seems like money well spent.

Proponents of buying put options in calm environments describe an opportunity to obtain protection at reduced prices during the calm before the storm. If history is any guide, the more likely outcome is that we are in the midst of the calm before the calm.

Global Evidence

We have presented evidence to show that low S&P 500 Index option prices have not indicated a good value. As a robustness exercise, we now test whether

Exhibit 10

Summary Statistics for 5% Out-of-the-Money Delta-Hedged Put Options on Global Equity Indexes, March 15, 1996, to June 30, 2014

For each index, the table shows the average ex post volatility risk premium and summary statistics for a long delta-hedged put backtest. The delta-hedged backtests referenced here bought 5% out-of-the-money front-month put options, sized to unit leverage, and held to expiry. Volatility risk premium defined as an index's volatility index minus its subsequent annualized volatility. The average volatility risk premium is computed as the square root of the average implied variance (the volatility index squared) minus the square root of the average coinciding realized variance.

For the volatility risk premium calculation, the end date is June 30, 2014, and the start dates are January 1996 for S&P 500, March 1996 for DAX, January 1999 for Euro Stoxx 50, June 1999 for Swiss Market Index, January 2000 for FTSE 100, January 2001 for Nikkei 225 and Hang Seng, February 2001 for NASDAQ 100, January 2003 for KOSPI 200, and January 2004 for Russell 2000. The volatility index used to bucket the S&P 500 is the VIX Index (CBOE Volatility Index). For DAX: V1X Index (Deutsche Borse VDAX-NEW Index). For Euro Stoxx: V2X Index (Euro Stoxx 50 Volatility Index). For FTSE: VFTSE Index (FTSE 100 Volatility Index). For Hang Seng: VHSI Index (HSI Volatility Index). For KOSPI: VKOSPI Index (KOSPI 200 Volatility Index). For NASDAQ: VXN Index (CBOE NASDAQ-100 Volatility Index). For Nikkei: VNKY Index (Nikkei Volatility Index). For Russell 2000: RVX Index (CBOE Russell 2000 Volatility Index). And for Swiss Market Index: V3X (Deutsche Borse VSMI Volatility Index).

For the long put option backtests, the end date is June 30, 2014, and the start dates are March 1996 for S&P 500, February 2001 for Nasdaq 100, January 2002 for Euro Stoxx 50, FTSE 100, DAX, and SMI, January 2004 for Russell 2000, May 2004 for KOSPI 200, June 2004 for Nikkei 225, and January 2006 for Hang Seng. The volatility indexes used are the same as for the volatility risk premium calculation.

Volatility, beta, and skew are computed using overlapping 21-day returns.

Average (Annualized)	S&P 500	DAX	Euro Stoxx 50	FTSE 100	Hang Seng	KOSPI 200	NASDAQ 100	Nikkei 225	Russell 2000	Swiss Market Index
Volatility Risk Premium	3.4%	2.4%	3.4%	2.9%	2.0%	2.3%	2.8%	2.6%	2.0%	2.3%
Annualized Excess Return	–2.0%	–1.9%	–2.2%	–2.2%	–2.6%	–1.7%	–1.9%	–2.2%	–3.2%	–1.3%
Annualized Volatility	2.4%	1.8%	2.1%	2.1%	3.1%	2.5%	2.7%	2.9%	2.3%	2.0%
Sharpe Ratio	–0.83	–1.09	–1.01	–1.09	–0.84	–0.68	–0.72	–0.75	–1.36	–0.63
Beta to S&P 500 Index	–0.03	0.00	–0.02	–0.03	–0.01	–0.03	–0.01	–0.03	–0.02	–0.02
- Upside Beta	0.01	0.02	0.02	–0.01	0.00	–0.01	0.02	0.01	0.03	0.01
- Downside Beta	–0.10	–0.03	–0.06	–0.08	–0.05	–0.09	–0.06	–0.10	–0.08	–0.06
Skew	6.0	0.4	1.5	2.8	–0.7	1.4	1.5	0.8	3.2	1.2

Sources: AQR, Option Metrics, Bloomberg.

this result holds over a larger universe of international equity indexes. Specifically, we analyze index options on the DAX, Euro Stoxx 50, FTSE 100, Hang Seng, KOSPI 200, NASDAQ 100, Nikkei 225, Russell 2000, and Swiss Market Index.[6]

Before bucketing by their respective implied volatility indexes, we begin by reporting their unconditional volatility risk premiums in Exhibit 10. Exhibit 10 also reports the return properties of delta-hedged 5% out-of-the-money put options in each index, which corresponds to the rightmost column of Exhibit 4. The properties of put options purchased on global equity indexes are not substantially different than put options purchased on the S&P 500. Option prices on each equity index reflect a volatility risk premium and buying put options for protection in any of these indexes is detrimental to average realized returns.

Exhibit 11 buckets into quintiles the ex post volatility risk premium in each index by its respective implied volatility indexes—a global version of Exhibit 6.[7] Exhibit 12 buckets into quintiles each index's delta-hedged put option returns by their respective implied volatility indexes—a global version of Exhibit 8. Both exhibits show that our results for the S&P 500 Index extend to other major global equity indexes.

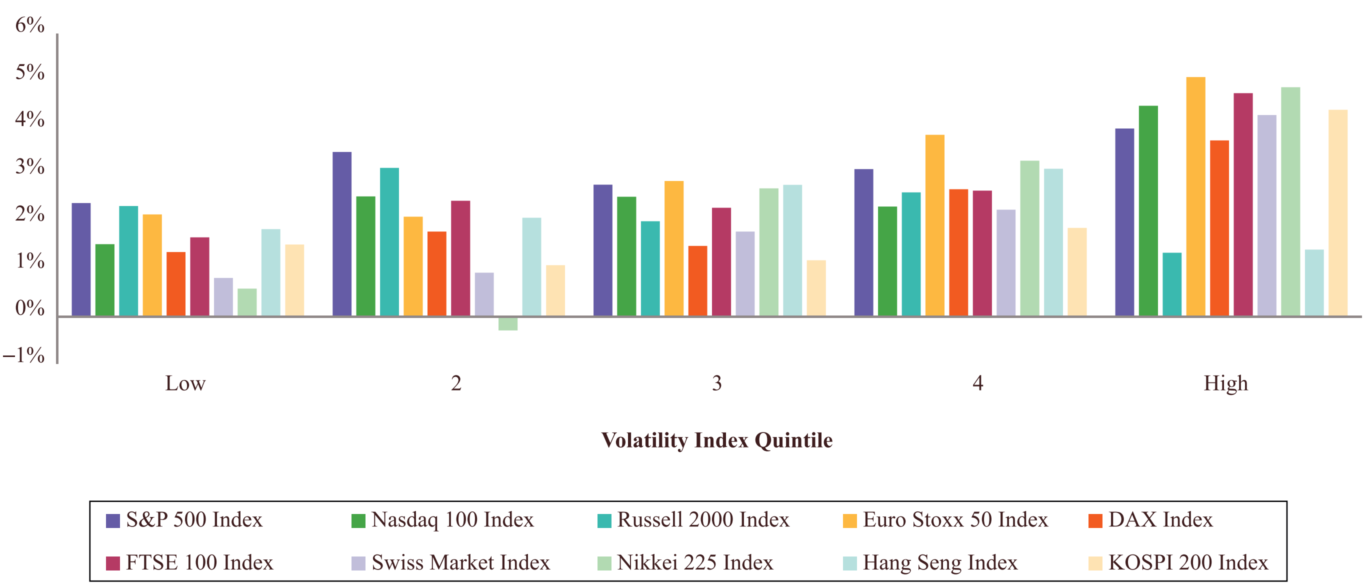
Exhibit 13 computes relevant statistics for each index in each bucket and then averages across indexes within buckets. Options include a volatility risk premium in each volatility bucket, including the calmest quintile environment. The difference between implied and realized volatility and the losses to purchasing put options increase with volatility. However, on a risk-adjusted basis, calm markets are not substantially different from other environments. The Sharpe ratio of

EXHIBIT 11

Ex Post Volatility Risk Premium (bucketed by local "VIX" index) on Global Equity Indexes, January 4, 1996, to June 30, 2014

Average ex post volatility risk premium in each quintile after bucketing by each index's volatility index at the beginning of the period (all volatility index sorts include a one-day lag to avoid overlapping data biasing results). Volatility risk premium defined as an index's volatility index minus its subsequent annualized volatility. The average volatility risk premium is computed as the square root of the average implied variance (the volatility index squared) minus the square root of the average coinciding realized variance.

For this calculation, the end date is June 30, 2014, and the start dates are January 1996 for S&P 500, March 1996 for DAX, January 1999 for Euro Stoxx 50, June 1999 for Swiss Market Index, January 2000 for FTSE 100, January 2001 for Nikkei 225 and Hang Seng, February 2001 for Nasdaq 100, January 2003 for KOSPI 200, and January 2004 for Russell 2000. The volatility indexes used are the same as for the volatility risk premium calculation reported in Exhibit 10.



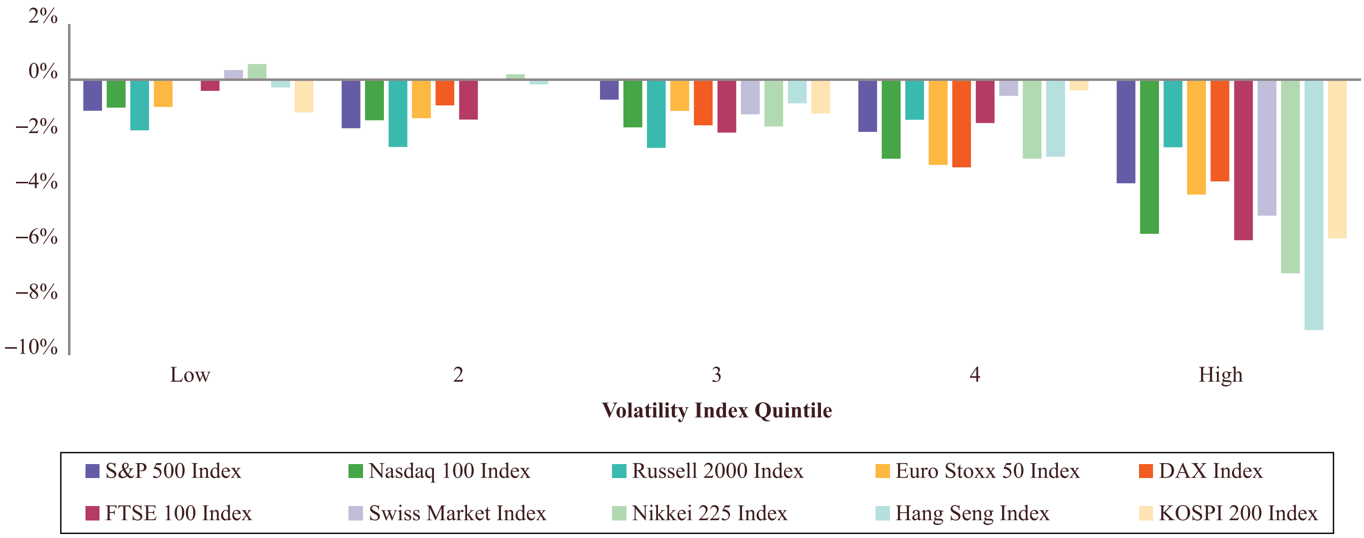
Sources: AQR, Bloomberg.

EXHIBIT 12

Long 1-Month Delta-Hedged 5% Out-of-the-Money Put Options Annualized Return (bucketed by VIX index) on Global Equity Indexes, March 15, 1996, to June 30, 2014

For each index, the chart shows the annualized average return for a long delta-hedged put backtest, where the returns are bucketed by quintile of that index's volatility index. The delta-hedged backtests referenced here bought 5% out-of-the-money front-month put options, sized to unit leverage, and held to expiry.

For this calculation, the end date is June 30, 2014, and the start dates are March 1996 for S&P 500, February 2001 for Nasdaq 100, January 2002 for Euro Stoxx 50, FTSE 100, DAX, and SMI, January 2004 for Russell 2000, May 2004 for KOSPI 200, June 2004 for Nikkei 225, and January 2006 for Hang Seng. The volatility indexes used are the same as for the volatility risk premium calculation reported in Exhibit 10.



Sources: AQR, Option Metrics, Bloomberg.

buying options (delta-hedged) is similar (–0.9) in all but the highest volatility quintile, where it is much worse. Similarly, the volatility risk premium when computed as a ratio is also similar across all five volatility buckets.

Our analysis of put options purchased on global equity indexes provides additional evidence in support of the following conclusion: Options have been expensive on average, and they have been expensive in different volatility environments. Simply put, we reject the argument that options are cheap in calm times.

Black Swans

The historical evidence presented thus far favors selling over buying options, even in calm periods. Those who favor buying options for protection may argue that history does not present all possible future outcomes. The options are not purchased to specifically protect against events witnessed in prior calm environments but to protect against a potential extreme negative event that has not yet occurred—the black swan.

We consider a hypothetical black swan event: the S&P 500 is down 20% in a day and implied volatility spikes to 150%, approximating what happened in the October 1987 crash. We compute the black swan return earned by 5% OTM put options with 30 days until expiration, purchased at the average implied volatility within each VIX Index decile and adjusted for the put option's delta just prior to the black swan event.[8] Exhibit 14 plots the black swan annual frequency required for put options to break even. We estimate this frequency by dividing the annualized return to holding the option as reported in Exhibit 8 by the put option's black swan return. These reported frequencies are for incremental black swan events above and beyond those already included in our sample.

On average, this hypothetical black swan event has to occur at least once every 10 years for put option purchases to break even. For the highest VIX decile, the black swan event must occur at least once every four years

EXHIBIT 13

Summary Statistics for 5% Out-of-the-Money Delta-Hedged Put Options—Equal-Weight Average within Buckets across Global Equity Indexes, March 15, 1996, to June 30, 2014

Statistics are computed by independently bucketing into quintiles country-by-country, computing the relevent metric, and then averaging across countries. "Implied—Realized" is defined as an index's volatility index minus its subsequent annualized volatility. For each index, the average "Implied—Realized" is the square root of the average implied variance (the volatility index squared) minus the square root of the average coinciding realized variance. For each index, the average Log(Implied/Realized Volatility) is the log of the ratio of the square root of the average implied variance to the square root of the average coinciding realized variance. The delta-hedged backtests referenced here bought 5% out-of-the-money front-month put options, sized to unit leverage, and held to expiry. Volatility is computed using overlapping 21-day returns.

For the volatility risk premium calculation, the end date is June 30, 2014, and the start dates are January 1996 for S&P 500, March 1996 for DAX, January 1999 for Euro Stoxx 50, June 1999 for Swiss Market Index, January 2000 for FTSE 100, January 2001 for Nikkei 225 and Hang Seng, February 2001 for Nasdaq 100, January 2003 for KOSPI 200, and January 2004 for Russell 2000. For the long put option backtests, the end date is June 30, 2014, and the start dates are March 1996 for S&P 500, February 2001 for Nasdaq 100, January 2002 for Euro Stoxx 50, FTSE 100, DAX, and SMI, January 2004 for Russell 2000, May 2004 for KOSPI 200, June 2004 for Nikkei 225, and January 2006 for Hang Seng. The volatility indexes used are the same as for the volatility risk premium calculation reported in Exhibit 10.

Average (Annualized)	S&P 500	DAX	Euro Stoxx 50	FTSE	Hang Seng	KOSPI	NASDAQ	Nikkei	Russell 2000	Swiss Market Index
Volatility Risk Premium	3.4%	2.4%	3.4%	2.9%	2.0%	2.3%	2.8%	2.6%	2.0%	2.3%
Annualized Excess Return	−2.0%	−1.9%	−2.2%	−2.2%	−2.6%	−1.7%	−1.9%	−2.2%	−3.2%	−1.3%
Annualized Volatility	2.4%	1.8%	2.1%	2.1%	3.1%	2.5%	2.7%	2.9%	2.3%	2.0%
Sharpe Ratio	−0.83	−1.09	−1.01	−1.09	−0.84	−0.68	−0.72	−0.75	−1.36	−0.63
Beta to S&P 500 Index	−0.03	0.00	−0.02	−0.03	−0.01	−0.03	−0.01	−0.03	−0.02	−0.02
Upside Beta	0.01	0.02	0.02	−0.01	0.00	−0.01	0.02	0.01	0.03	0.01
Downside Beta	−0.10	−0.03	−0.06	−0.08	−0.05	−0.09	−0.06	−0.10	−0.08	−0.06
Skew	6.0	0.4	1.5	2.8	−0.7	1.4	1.5	0.8	3.2	1.2

Sources: AQR, Option Metrics, Bloomberg.

to break even. For the lowest VIX decile, the black swan event would need to occur at least once every 21 years.

The last time an event this magnitude occurred was more than 27 years ago, in October 1987. Memory is a funny thing. Many people recall the crash as a complete shock within calm markets. In fact, the lowest implied volatility (as measured by the VXO Index) over the prior month was 21.2%, placing the environment preceding the crash in the 7th VIX Index decile as shown in Exhibit 5. Within this decile, the October 1987 crash would need to occur once every 10 years for purchasing put options to break even.

However, the October 1987 event is the worst daily crash on record for the S&P 500 going back to 1950. The next three largest daily events all occurred in 2008 (October 15, December 1, and September 29) and are thus are already included in the analysis in preceding sections. In each of these events, the S&P 500 was down nearly 10%.[9]

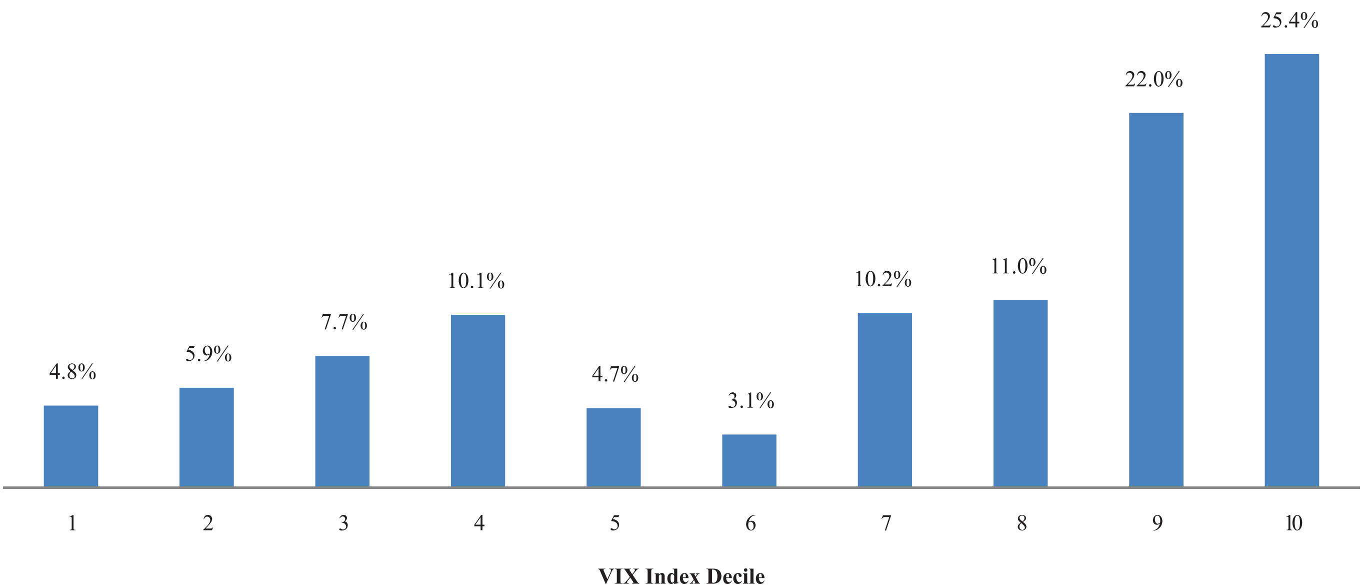
If you believe that the type of black swan event considered in this section is *significantly* under-represented in our historical record and you are also willing to pay out more than 1% of NAV per year in order to buy protection for such an event, then purchasing put options may be rationalized. And while it is certainly possible that black swans are under-represented and put options are less expensive than they appear or are even cheaply priced, we should similarly be willing to also entertain the possibility that black swan events are over-represented in our sample (occurring less often than once every 64 years) and put options are even more expensive than they appear.

However, if an investor has a belief that black swan events are under-represented in historical observations, it seems natural that this belief should be reflected not only in the purchasing of options but also in the direct asset allocation to equities. Purchasing options is an indirect way of reducing equity exposure and more direct solutions should be considered as well—in particular those that may earn a positive expected return, such as fixed income and uncorrelated alternatives. These more "permanent" solutions have the benefit of increasing diversification while not requiring market timing ability or dependence on some illusory good deal in options markets. To the extent option markets do offer a good deal occasionally, investors can still take advantage of that opportunistically.

Exhibit 14

Black Swan Annual Frequency Required for 5% Out-of-the-Money Put Options to Break Even (bucketed by the VIX index), March 15, 1996, to June 30, 2014

The chart shows the black swan annual frequency required for a long delta-hedged put backtest to break even, bucketed by decile of the VIX. The delta-hedged backtest referenced here bought 5% out-of-the-money front-month S&P 500 put options, sized to unit leverage, and held to expiry. Average annualized returns were computed within each bucket. Separately, the average instantaneous return due to the S&P 500 declining by 20% and its implied volatility increasing to 150% for one-month delta-hedged 5% out-of-the-money put options was computed for each decile. The annual frequency is computed by dividing the first number by the second and negating.



Sources: AQR, Option Metrics, Chicago Board Options Exchange, Standard and Poor's.

CONCLUSION

Our analysis demonstrates that put options' low prices during calm periods give the illusion of value. Buying an option is not a bet that realized volatility will increase; it is a bet that realized volatility will increase above the option's implied volatility. Buying an option is expected to lose money even when volatility is low and rising if the spread between realized and implied volatility is sufficiently high.

The possibility of black swan events is an often-quoted justification for the large observed volatility risk premium. We believe the frequency of black swan events required to rationalize option purchases is unreasonably large given our knowledge of extreme events. More importantly, we believe investors are best served by integrating their beliefs regarding black swans to their aggregate asset allocation as opposed to opportunistically purchasing portfolio insurance at low, but not cheap, prices. This is especially the case for the majority of investors who have limited ability to stick with a hedging program that loses money for years while awaiting an episodic payoff.

In conclusion, equity index options have been an expensive form of portfolio protection, even when their prices were low.

ENDNOTES

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[1]The VIX Index is the S&P 500 Index's volatility implied by option prices. A low VIX tells us option prices are also low.

[2]A long option position is considered to be long volatility because the option's price is positively related to expected volatility and the profitability of owning the option, after

adjusting for its equity exposure, is positively related to the equity's realized volatility.

[3]The average volatility risk premium is computed as the square root of the average implied variance (VIX squared) minus the square root of the average coinciding realized variance. If computed as the average VIX minus average realized volatility, the 4.4% estimate has a positive bias.

[4]Bakshi and Kapadia [2003] show that delta-hedged option returns exhibit a volatility risk premium. Bollen and Whaley [2004] find that the shape of the implied volatility surface is influenced by net option buying pressure. Similarly, Garleanu et al. [2005] present theoretical and empirical evidence in favor of option demand pressure explaining the volatility risk premium. Hill et al. [2006] and Israelov and Nielsen [2014] show that the short volatility exposure embedded in covered call strategies enhance their risk-adjusted returns.

[5]This article investigates the relationship between the volatility risk premium and market risk by sorting on the VIX Index. However, the VIX Index is not a pure measure of risk because it includes a volatility risk premium. For robustness, we have also performed the same analysis after sorting on historical realized volatility (42-day) instead. Due to the strong relationship between the VIX Index and realized volatility, the results of the two sorts are similar and any conclusions drawn from the analysis presented in the article are robust to bucketing by realized volatility rather than by the VIX Index.

[6]NASDAQ enters our sample in February 2001. DAX, Euro Stoxx, FTSE, and the Swiss Market Index join in January 2002. The Russell 2000 is available beginning January 2004. KOSPI joins in May of 2004 and Nikkei joins in June 2004.

[7]The volatility index used to bucket for the S&P 500 is the VIX Index (CBOE Volatility Index). For DAX: V1X Index (Deutsche Borse VDAX-NEW Index). For Euro Stoxx: V2X Index (Euro Stoxx 50 Volatility Index). For FTSE: VFTSE Index (FTSE 100 Volatility Index). For Hang Seng: VHSI Index (HIS Volatility Index). For KOSPI: VKOSPI Index (KOSPI 200 Volatility Index). For NASDAQ: VXN Index (CBOE NASDAQ-100 Volatility Index). For Nikkei: VNKY Index (Nikkei Volatility Index). For Russell 2000: RVX Index (CBOE Russell 2000 Volatility Index). And for Swiss Market Index: V3X (Deutsche Borse VSMI Volatility Index). We bucket into quintiles rather than deciles because of the reduced sample size.

[8]This delta adjustment is important because the option premium is paid for the option's convexity and not for its delta. Portfolio managers may reduce their equity exposure directly by selling SPX futures; there is no need for options simply to reduce equity exposure.

[9]Interestingly, in each of these three instances, the S&P 500 Index had moved significantly after the option was purchased, but before the event. As a result, the put options' convexity on the S&P 500 crash dates was quite low. Even though the index was down nearly 10% in each case, the delta-adjusted put options returned less than 1% during each event.

REFERENCES

Bakshi, G., and N. Kapadia. "Delta-Hedged Gains and the Negative Market Volatility Risk Premium." *Review of Financial Studies*, Vol. 16, No. 2 (2003), pp. 527-566.

Bollen, N.P.B., and R.E. Whaley. "Does Net Buying Pressure Affect the Shape of Implied Volatility Functions?" *Journal of Finance*, Vol. 59, No. 2 (2004), pp. 711-753.

Gârleanu, N., L.H. Pedersen, and A.M. Poteshman. "Demand-Based Option Pricing." *Review of Financial Studies*, Vol. 22, No. 10 (2009), pp. 4259-4299.

Hill, J.M., V. Balasubramanian, K. Gregory, and I. Tierens. "Finding Alpha via Covered Index Writing." *Financial Analysts Journal*, Vol. 62, No. 5 (2006), pp. 29-46.

Israelov, R., and L. Nielsen. "Covered Call Strategies: One Fact and Eight Myths." *Financial Analysts Journal*, Vol. 70, No. 6 (2014), pp. 23-31.

——. "Covered Calls Uncovered." AQR working paper, (2015).

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