FAJ
Financial Analysts Journal
Volume 70 • Number 1
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The Limits to Arbitrage and the Low-Volatility Anomaly

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The authors found that over 1963–2010, the existence and trading efficacy of the low-volatility stock anomaly were more limited than widely believed. For example, they found no anomalous returns for equal-weighted long–short (low-risk minus high-risk) portfolios and that alpha is largely eliminated when omitting low-priced stocks from value-weighted long–short portfolios. Furthermore, performance of long–short portfolios was significantly reduced by high transaction costs, reflecting the finding that the abnormal returns were concentrated among low-liquidity and smaller stocks. Amplifying liquidity needs, the anomalous excess returns quickly reversed, requiring frequent rebalancing. The authors' findings have meaningful implications for implementing low-risk equity portfolio strategies.

In what is sometimes referred to as the "low-risk" or "low-volatility" anomaly, researchers have discovered a provocative empirical inverse connection between future stock returns and various measures of stock return variability, including total return volatility, idiosyncratic volatility, and beta. Documented empirical evidence has shown that future stock returns of low-return-variability portfolios outperform those of high-return-variability portfolios in both US and international markets (see, e.g., Ang, Hodrick, Xing, and Zhang 2006, 2009; Blitz and van Vliet 2007; Clarke, de Silva, and Thorley 2010; Baker, Bradley, and Wurgler 2011; Frazzini and Pedersen, forthcoming; Li, Sullivan, and Garcia-Feijóo, forthcoming 2015). These findings run counter to our economic intuition because economic theory predicts that higher expected return compensates for higher expected risk.

Garcia-Feijóo, Kochard, Sullivan, and Wang (2013) suggested that the so-called low-risk anomaly might more accurately be called the *high-risk* anomaly given that the anomalous historical returns are found primarily among stocks in the highest-risk quintile. They demonstrated that the historical performance of low-risk investing is strikingly cyclical, driven largely by swings in the relative valuation levels of low-risk and high-risk stocks and by the varying investor appetite for momentum-driven investing. Taken together, these findings motivated us to further investigate the properties of this anomalous low-volatility effect.

In our study, we explored anomalous "low-risk" returns to better understand the underlying economics of low-volatility stock outperformance. In particular, we explored whether the abnormal returns associated with the persistent low-volatility anomaly can be captured over time in practice or are somehow subsumed by limits to investors' arbitraging them away. To accomplish this objective and gain additional insight into the seemingly powerful association between historical volatility and future returns, we examined the role of portfolio rebalancing and transaction costs in the persistence of the low-risk anomaly and in attempts to extract profits from the anomaly. Specifically, we examined portfolio-rebalancing requirements and the impact of associated transaction costs on the returns to zero-cost (low-risk minus high-risk) portfolios formed on the basis of various low-volatility measures.

Discussion of findings. We found that investors' ability to extract the excess returns of zero-cost portfolios that are based on both idiosyncratic volatility (IVOL) and the estimated CAPM (capital asset pricing model) beta is limited—likely a surprising result for many observers. In particular, we found that the excess returns of the low-risk, zero-cost portfolios are short lived because they are present only in month $t + 1$ and are largely subsumed by

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Editor's Note: Rodney N. Sullivan, CFA, is editor of the *Financial Analysts Journal*. He was recused from the referee and acceptance processes and took no part in the scheduling and placement of this article. See the *FAJ* policies section of cfapubs.org for more information.

high transaction costs. Moreover, we found that the anomalous returns of value-weighted portfolios are largely eliminated when low-priced (under $5) stocks are omitted—and are completely absent from equal-weighted portfolios. Our results are based on a series of tests in which we separated the universe of US stocks into high- and low-liquidity segments and then related the performance of these liquidity segments to the zero-cost, IVOL-based portfolio. In conducting this analysis, we determined whether the abnormal returns of our zero-cost, volatility-based portfolios persist beyond the first month and whether the excess returns are concentrated in the low-liquidity segment. Such a finding would imply that traders will necessarily experience relatively high transaction costs in attempting to extract excess returns.

The Low-Volatility Anomaly

Of the many possible approaches to measuring the low-volatility effect, the literature follows two primary paths: one that uses one month of daily returns (Ang et al. 2006, 2009) and one that uses a longer period (36 months or 60 months) of monthly returns (Clarke et al. 2010). Risk is typically measured by either idiosyncratic volatility or beta. In our study, we used all the various low-volatility measures in testing the efficacy of investors' attempts to arbitrage the low-volatility effect away.

We first explored the low-volatility trading strategy as used by Ang et al. (2006, 2009), who found that stocks with high IVOL in one month have low returns the next month. In discovering this one-month volatility effect, Ang et al. (2006, 2009) formed value-weighted portfolios by buying low-IVOL stocks and selling high-IVOL stocks (based on their IVOL ranking in the prior month) and holding the portfolios for one month. Their portfolios generated statistically significant excess returns over time, and these returns held when controlling for a variety of factors, including the well-known size and style effects of Fama and French (1993).[1] We tested this portfolio trading strategy with both value-weighted and equal-weighted portfolios. We then tested the strategy by using idiosyncratic volatility estimates over 36 months (IVOL36) and 60 months (IVOL60) as well as holding the portfolios for one month. Finally, we also formed portfolios by assigning stocks on the basis of the magnitude of their estimated CAPM beta (BETA).

Researchers' curious findings about volatility return patterns have spurred important research contributions aimed at a better understanding of the low-volatility anomaly. For example, in implementing a trading strategy, traders must obviously consider available liquidity. In our study, we attempted to ascertain the extent to which liquidity affects the viability of implementing various low-volatility trading strategies.[2] We also tried to determine whether the IVOL anomaly derives from investor mispricing or from systematic market risk—an important distinction for investors. Should the anomaly be related to systematic risk, then the excess returns could be viewed as arising from some, as yet unknown, common risk factor(s). For instance, Merton (1987) offered an explanation for why investors would demand higher returns for taking on higher IVOL. He explained that IVOL is positively related to expected return when investors cannot fully diversify their portfolios. Therefore, investors demand a higher return from companies with higher IVOL to compensate for imperfect diversification.

Ang et al. (2009) found that the IVOL anomaly extends to numerous countries and is highly correlated with that found in the United States. They argued that such an effect could be driven by latent systematic risks. Specifically, they showed that abnormal returns generated by IVOL-based portfolio strategies in international markets strongly co-move with those in the US markets, suggesting a common risk factor: "The large commonality in co-movement . . . suggests that broad, not easily diversifiable factors lie behind this effect" (p. 2). The co-movement finding implies that the return-predictive power of idiosyncratic risk is likely due to some pervasive risk factor. Similarly, Clarke et al. (2010) argued that IVOL (and total volatility) should be considered an additional equity market risk factor that investors should incorporate into portfolio construction. Interestingly, the empirical evidence in Ang et al. (2009) and Clarke et al. (2010) runs counter to the prediction by Merton (1987).

The excess return, however, may be unrelated to systematic risk. Instead, it may be driven by a mispricing—perhaps associated with an imperfection such as investor irrationality, perhaps somehow connected with IVOL. In exploring the competing risk versus mispricing explanations of the low-volatility anomaly, Li, Sullivan, and Garcia-Feijóo (forthcoming 2015) traced the observed low-volatility return's link to market mispricing. Those findings suggest that the highly anomalous returns to IVOL portfolios, as identified in the literature, cannot be viewed as compensation for factor risk. In the case of mispricing, the profit opportunity may be ephemeral as investors come to understand their cognitive error and arbitrage away any excess return. Or the mispricing could be more lasting, supported over time by the high costs of arbitraging away the anomalous returns and perhaps even by some behavioral considerations.

Focusing on market beta, Black (1972) offered a theoretically consistent interpretation of why low-risk stocks do so well relative to high-risk stocks. He showed that such borrowing restrictions as margin requirements may cause low-beta stocks to outperform. More recently, Baker et al. (2011) proposed that behavioral considerations, including the structure of institutional client mandates, discourage arbitrage activity that would otherwise eliminate the low-volatility effect. These two explanations may provide insight into why the low-volatility effect has persisted for so many years despite being highly recognized.

Data and Sample

We obtained our stock return data from the CRSP monthly stock return files for July 1963 through December 2010. For delisted companies, the CRSP monthly return file does not include the returns from the delisting month unless the delisting date is at the end of the month. We fetched the returns in the delisting month and the market capitalization on the delisting date from the CRSP daily return files and combined those returns with the delisting returns to create the effective delisting month returns. If the delisting was for performance-related reasons, however, we set the delisting return equal to –55% for trading on NASDAQ or –30% for the NYSE and Amex.

We concentrated on measuring idiosyncratic risk with respect to the Fama–French three-factor model. Specifically, we measured IVOL as the standard deviation of the residuals from the Fama–French three-factor model by regressing the daily returns of individual stocks in excess of the one-month T-bill rate, $R_{i,t} - R_{f,t}$, on the daily returns to the common factors of size and book-to-market ratio. In other words, for each stock i, we performed the following time-series regression:

$$R_{i,t} - R_{f,t} = a_i + b_i\left(R_{M,t} - R_{f,t}\right) + s_i SMB_t + h_i HML_t + \varepsilon_{i,t}, \tag{1}$$

where $R_{M,t} - R_{f,t}$, SMB, and HML represent the Fama–French market, size, and value factors, respectively. Following Ang et al. (2006), we required a minimum of 17 observations for model estimation. With this requirement, we omitted the most illiquid stocks from our results, thus minimizing the likelihood that our results are biased toward stocks that trade infrequently.

We measured IVOL36 and IVOL60 similarly as the standard deviation of the residuals from regressions of monthly excess returns on the Fama–French three-factor model, using 36 months (or at least 12, depending on availability) or 60 (at least 24). We further measured low risk as the estimated beta (b_i) from the CAPM market model:

$$R_{i,t} - R_{f,t} = a_i + b_i\left(R_{M,t} - R_{f,t}\right) + \varepsilon_{i,t}, \tag{2}$$

where the dependent variable is the monthly excess return of an equal- or value-weighted portfolio. Beta is computed monthly by regressing excess returns on the CRSP value-weighted index over the prior 60 months (a minimum of 24 months).

Measures of Volatility and Liquidity

As a tool to better gauge the data in our analysis, **Table 1** provides a summary of distribution and correlation statistics for the key variables. To examine trading strategies based on low volatility, we began by forming quintile portfolios on the basis of the level of IVOL, which allowed us to explore the average return variation among the various IVOL quintile portfolios, including a zero-cost IVOL portfolio (lowest-IVOL quintile minus highest-IVOL quintile). We tested the performance of these portfolios over periods subsequent to their formation. To accomplish this task, we applied a commonly used rank portfolio test while controlling for the well-known Fama–French (1993) effects of size and style.

In particular, our portfolio formation strategy was based on an estimation period of N months (for our purposes, we focused on $N = 1$, 36, and 60), and we held these value-weighted portfolios over M months. At month t, we computed risk from regressions (Equation 1 and Equation 2) on data over the previous N months. We then constructed value-weighted and equal-weighted portfolios on the basis of these risk measures and held these portfolios for up to 12 months. Because we found that the zero-cost IVOL portfolio is meaningfully profitable only in the first month (discussed later), we concentrated the bulk of our analysis on the one-month holding period strategy ($M = 1$), whereby we simply sorted stocks into quintile portfolios on the basis of their level of IVOL, IVOL36, IVOL60, or BETA and held the portfolios for one month. The portfolios were rebalanced each month.

We first report the average intercept (alpha) results from our Fama–French (1993) three-factor quintile regression portfolios, as sorted on IVOL ($N = 1$), IVOL36 ($N = 36$), and IVOL60 ($N = 60$) and adjusted for the effects of size and style. The dependent variables are the IVOL-based portfolio returns in excess of the risk-free rate.[3]

Using several empirical measures of liquidity, we explored the efficacy of the IVOL trading strategy (see Stoll 2000). For our first measure, we used the average stock price per share in the month prior to portfolio formation. It is well known that stocks with a market value of less than $5 a

Table 1. Summary Statistics, July 1963–December 2010

	IVOL Daily (%)	IVOL36 (%)	IVOL60 (%)	BETA	Average Price ($)	Average NonZEROR (%)	Average Amihud	Average DVOL ($ millions)
Variable								
Mean	14.38	12.97	12.92	1.17	23.86	80.56	6.54	179.97
Std. dev.	14.52	8.84	8.28	0.80	658.47	20.00	131.71	2,875.00
Quartile 3	17.65	15.99	15.98	1.57	24.73	95.24	1.22	26.14
Median	10.58	10.76	10.87	1.10	13.02	85.71	0.14	3.52
Quartile 1	6.36	7.34	7.48	0.68	5.35	71.43	0.01	0.55
Correlation								
IVOL Daily	1.00							
IVOL36	0.48	1.00						
IVOL60	0.46	0.94	1.00					
BETA	0.10	0.28	0.31	1.00				
Price	–0.01	–0.02	–0.02	–0.09	1.00			
NonZEROR	–0.03	–0.11	–0.12	0.10	0.02	1.00		
Amihud	0.19	0.06	0.06	–0.01	0.00	–0.04	1.00	
DVOL	–0.02	–0.04	–0.04	0.02	0.01	0.05	0.00	1.00

share—so-called penny stocks—possess meaningfully less liquidity for trading purposes and are subject to market manipulation versus nonpenny stocks (see, e.g., Bradley, Cooney, Dolvin, and Jordan 2006).

For our second liquidity measure, we proxied transaction costs (e.g., bid–ask spread and commissions) as the proportion of trading days with nonzero returns (see Lesmond, Ogden, and Trzcinka 1999).[4] We measured the incidence of nonzero returns as the percentage of days in the prior month that a stock has a nonzero return (NonZEROR). The idea behind NonZEROR is that a security with high transaction costs—and thus low liquidity—will have less price movement and more zero-return days than a security with low transaction costs and high liquidity. The underlying premise is that the marginal trader will trade only when the marginal benefit of the information signal exceeds the costs of trading; otherwise, the security will have a zero return. In short, NonZEROR takes on smaller values for lower levels of liquidity.

For our third liquidity measure, we used dollar volume (DVOL), defined as the product of the daily closing share price and share volume, to proxy for the ease with which arbitrageurs can accumulate and liquidate trading positions. Higher values of DVOL suggest greater amounts of available liquidity, meaning that arbitrageurs can more easily transact.

Finally, we applied the Amihud (2002) illiquidity measure, which is calculated for each stock i in every month as follows:

$$\text{Amihud}_{it} = \frac{1}{t}\sum_{t}\frac{1,000,000\times\left|\text{Return}_t\right|}{\text{Price}_t\times\text{Volume}_t}, \tag{3}$$

where t is a positive-volume trading day in the month the measure is calculated. Note that a higher value for the Amihud illiquidity measure signifies higher illiquidity because a particular dollar volume traded is associated with a relatively strong price movement. We calculated all our measures of expected transaction costs during the same month in which we measured IVOL or BETA.

Table 1 presents summary statistics for our measures of volatility (IVOL, IVOL36, IVOL60, and BETA) and our measures of liquidity (average price, DVOL, Amihud, and NonZEROR). The daily IVOL is multiplied by the square root of the number of trading days in a month to make it comparable to the other (monthly) volatility measures.

As Table 1 shows, in comparing each measure of volatility, we can see large changes in the level of volatility, with Quartile 3's volatility approximately double that of Quartile 1. Likewise, in comparing each liquidity metric for Quartiles 1 and 3, we see meaningful changes in each liquidity level. In the bottom portion of Table 1, we can see that the average correlations between our four liquidity metrics are quite low, with near-zero correlations between our various measures of volatility and all our liquidity metrics.

Estimating Low-Risk Anomalous Returns

Table 2 reports the average alpha of each quintile portfolio, indicating the return-predictive power of IVOL for various months following portfolio formation. Column 1 shows the value-weighted alpha for month $t + 1$ over the full sample period, 1963–2010. We can see that the stocks in the highest-ranked IVOL quintile portfolio underperform all other month $t + 1$ quintile portfolios. Interestingly, this average alpha is relatively unchanged for the three lowest-ranked risk quintiles and declines for the two highest-ranked risk quintiles. The underperformance of that highest-ranked IVOL portfolio is meaningful compared with the three lowest-ranked IVOL quintiles, each posting near-zero average excess returns—all statistically insignificant from zero. Although the average alpha of Quintile 4, at –0.30%, is lower than that of the three lowest-IVOL quintiles, it is still higher than that of the highest-IVOL quintile. These results are consistent with the IVOL anomaly—the notion that portfolios formed with low-IVOL stocks outperform their high-IVOL counterparts in the following month.

We then focused our attention on the more interesting zero-cost quintile spread portfolio, which seeks to arbitrage the difference between the lowest-ranked and highest-ranked quintiles. As shown in Table 2 (last row of column 1), the factor-adjusted, zero-cost portfolio has a statistically significant average alpha of 1.19% for 1963–2010, suggesting that current-month IVOL has a significant negative relationship with the one-month post-formation ($t + 1$) return. These results again demonstrate that current-month low-IVOL stocks outperform high-IVOL stocks in the following month, a finding consistent with Ang et al. (2006, 2009).

Interestingly, however, as shown in columns 2–4 of Table 2, the observed outperformance monotonically declines during succeeding months after portfolio formation. That is, in extending our tests to include holding periods beyond the first two months following portfolio formation, we found that current-month IVOL has no meaningful relationship with stock returns in periods beyond the second month.[5] Empirically then, the excess returns of the IVOL anomaly all occur in the first month or two following portfolio formation.

Table 2. Abnormal Returns for IVOL-Based Portfolios: Monthly Regressions of IVOL Quintile Portfolio Returns, July 1963–December 2010

	Month t + 1	Month t + 2	Month t + 3	Month t + 12
Rank	(1)	(2)	(3)	(4)
1	0.08*	0.46***	0.52***	0.37**
	(1.87)	(2.60)	(3.04)	(2.10)
2	0.07	0.50**	0.62***	0.44**
	(1.13)	(2.30)	(2.82)	(2.09)
3	0.09	0.54**	0.64**	0.55**
	(1.09)	(1.98)	(2.46)	(2.11)
4	–0.30**	0.32	0.63**	0.60*
	(–2.35)	(0.94)	(1.96)	(1.92)
5	–1.11***	–0.31	0.13	0.30
	(–5.90)	(–0.80)	(0.35)	(0.83)
1 – 5	1.19***	0.77**	0.39	0.06
	(6.04)	(2.54)	(1.22)	(0.21)

Notes: Quintile 1 (5) corresponds to the quintile companies with the lowest (highest) IVOL characteristic; 1 – 5 is the difference portfolio between the lowest- and highest-ranked quintile portfolios, or the quintile spread portfolio. Returns are expressed in percentages. Heteroskedasticity-consistent t-statistics (White 1980) are in parentheses.

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

In short, we found that the IVOL effect is short lived, effectively requiring traders to adjust portfolio holdings at least every other month to have a reasonable chance at producing alpha. Such frequent rebalancing naturally raises questions about the impact of transaction costs and liquidity constraints. Our results have thus far ignored the impact of these crucially important components on the success of any trading strategy.

We then analyzed this issue in some detail. Our results, presented next, led us to conclude that transaction costs and liquidity constraints create significant barriers to traders' ability to effectively implement an IVOL-based trading strategy. In particular, we found that the various IVOL-based trading strategies we explored are profitable only for those subsamples in which transaction costs and liquidity constraints are significantly greater.

We report our zero-cost portfolio results for each quintile portfolio, indicating the return-predictive power of our three definitions of risk for various months following portfolio formation. **Table 3** shows the alpha results for various holding period months, $t + M$, where M = 1, 2, 3, or 12, as well as for our three definitions of risk and for both value-weighted and equal-weighted portfolios. Results are for the full sample period, 1963–2010.

Table 4 reports zero-cost portfolio results for various sample periods, different definitions of risk, and both value-weighted and equal-weighted portfolios. As shown in Tables 3 and 4, we draw conclusions similar to those discussed earlier concerning these various dimensions. Specifically, several noticeable trends emerge from an exploration of the data. First, results for average alpha are somewhat weaker with equal-weighted portfolios, a finding consistent across all periods and risk definitions.

Second, Table 3 reports results for holding periods of up to 12 months, showing that the observed zero-cost alpha monotonically declines rapidly in months following portfolio formation for all measures of risk: Current-month risk has no meaningful relationship with stock returns in periods beyond the second month. On the basis of this finding and our other findings discussed herein, we chose to focus on understanding transaction costs for only the t + 1 period in conducting additional analyses in our study.

Third, our alternative definitions of volatility (IVOL36 and BETA) generally present somewhat reduced average alphas versus IVOL for both value-weighted and equal-weighted portfolios.[6]

Fourth, the low-risk effect has been noticeably weaker since 1990, and the global financial crisis in 2008 had little impact on our long-term findings—that is, the average alpha results for 1991–2007 and 1991–2010 are quite similar, except for value-weighted returns based on IVOL. Our finding of weaker results after 1990 could be a result of improved market efficiency, reflecting

Table 3. Abnormal Returns for Low-Risk-Based Portfolios: Monthly Regressions of Zero-Cost Quintile Portfolio Returns for Month *t* + *M*, July 1963–December 2010
(*t*-statistics in parentheses)

	Zero Cost		Zero Cost (*P* < $5 excluded)	
Month *t* + *M*	Equal Weighted	Value Weighted	Equal Weighted	Value Weighted
IVOL				
t + 1	0.40*	1.19***	–0.56***	0.38**
	(1.88)	(6.04)	(–3.79)	(2.47)
t + 2	0.22	0.77**	–0.73***	0.04
	(0.77)	(2.54)	(–3.50)	(0.17)
t + 3	–0.09	0.39	–0.82***	–0.27
	(–0.29)	(1.22)	(–3.98)	(–1.05)
t + 12	–0.48	0.06	–0.81***	–0.40
	(–1.52)	(0.21)	(–4.77)	(–1.68)
IVOL36				
t + 1	0.38*	0.82***	–0.76***	–0.02
	(1.82)	(4.56)	(–5.48)	(–0.11)
t + 2	0.12	0.46	–0.94***	–0.32
	(0.41)	(1.51)	(–3.95)	(–1.18)
t + 3	–0.13	0.26	–0.98***	–0.45
	(–0.38)	(0.80)	(–4.07)	(–1.65)
t + 12	–0.25	0.36	–0.81***	–0.29
	(–0.72)	(1.15)	(–3.97)	(–1.09)
BETA				
t + 1	0.32**	0.25	–0.30**	0.11
	(2.02)	(1.43)	(–2.16)	(0.65)
t + 2	–0.00	0.03	–0.47*	–0.10
	(–0.01)	(0.09)	(–1.91)	(–0.33)
t + 3	–0.11	–0.18	–0.54**	–0.23
	(–0.39)	(–0.55)	(–2.26)	(–0.78)
t + 12	0.21	0.16	–0.16	0.04
	(0.81)	(0.49)	(–0.77)	(0.15)

Note: See notes to Table 2.
*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

the impact of regulations, passed in 1990, aimed at reducing fraud in the trading of penny stocks (Bradley et al. 2006).

In light of the weaker results since 1990, Table 3 also reports results for excluding stocks priced under $5 (*P* < $5 excluded). When penny stocks (a proxy for liquidity) are excluded, portfolio alpha is meaningfully reduced across the board—that is, for all risk measures, formation periods, and holding periods. We will return to the impact of penny stocks and, more generally, liquidity later in the article; however, it is important to note that our initial findings support the notion that stock illiquidity has important consequences for reported zero-cost alphas.

Estimating Arbitrage Costs for the IVOL Anomaly

We then more carefully examined whether the observed profitability of the IVOL-based strategy is subsumed by the impact of transaction costs imposed by liquidity constraints.

Columns 2–5 of **Table 5** report the average value of each of our liquidity measures as calculated in accordance with the ranking of each IVOL quintile portfolio. For three of our four liquidity measures, the liquidity level of the highest-volatility portfolio is meaningfully lower than that of the portfolios with low levels of volatility. The only exception,

Table 4. Abnormal Returns for Low-Risk-Based Portfolios, Various Periods: Monthly Regressions of Zero-Cost Quintile Portfolio Returns for Month *t* + 1
(*t*-statistics in parentheses)

	Equal Weighted	Value Weighted
IVOL		
1963–2010	0.40*	1.19***
	(1.88)	(6.04)
1963–1990	0.76***	1.44***
	(4.24)	(9.39)
1991–2010	0.05	1.02**
	(0.11)	(2.56)
1991–2007	–0.15	0.79
	(–0.29)	(1.68)
IVOL36		
1963–2010	0.38*	0.82***
	(1.82)	(4.56)
1963–1990	0.57***	1.04***
	(2.89)	(6.07)
1991–2010	0.30	0.70**
	(0.80)	(2.34)
1991–2007	0.25	0.62*
	(0.61)	(1.81)
BETA		
1963–2010	0.32**	0.25
	(2.02)	(1.43)
1963–1990	0.54***	0.46**
	(3.89)	(2.45)
1991–2010	0.15	0.07
	(0.50)	(0.24)
1991–2007	0.32	–0.03
	(1.01)	(–0.10)

Note: See notes to Table 2.

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

NonZEROR, shows that the proportion of zero returns is similar across IVOL quintiles. This result is unsurprising given that we required at least 17 days of nonzero returns to compute a stock's IVOL. Overall, our findings suggest a close, positive association between IVOL and illiquidity. Interestingly, we found an average price of $7.03 for the stocks in the highest-volatility quintile, which suggests that many, if not most, stocks in that portfolio would be considered penny stocks.

Given that many stocks in the highest-volatility quintile appear to be penny stocks, we next examined the average alpha of the IVOL quintile portfolios that exclude stocks with an average share price of less than $5 in the month prior to portfolio formation. We then studied the impact of liquidity on the universe of nonpenny stocks over the full sample period (1963–2010). The results reported in column 1 of **Table 6** indicate that when penny stocks are excluded, the average alpha for the zero-cost portfolio declines meaningfully to 0.38% from its earlier average value of 1.19%. The average values for the liquidity measures across quintiles, shown in columns 2–5 of Table 6, are similar to those found in Table 5, with the highest-volatility quintile exhibiting the poorest overall relative liquidity. However, liquidity is meaningfully improved across all IVOL quintiles except those that include stocks selling for less than $5 a share.

Table 7 presents the IVOL quintile rank portfolio test, with the results separated into liquidity tercile subsamples (low, mid, high) in accordance with the DVOL liquidity measure. For this analysis, we sorted stocks into terciles (low, mid, high) on the basis of DVOL liquidity. Columns 2–3 of Table 7 show that no profitability for the low-IVOL-based trading strategy can be found in the universe of low- and mid-liquidity stocks with prices above $5 because average excess returns are little changed with increasing volatility. Average excess returns for the highest-liquidity group (column 4) show that the quintile spread portfolio yields a significant 0.45% average excess return for value-weighted portfolios.[7] Thus, column 4 shows that IVOL-related abnormal returns for value-weighted portfolios with sufficient liquidity for trading decline by about 60%—that is, alpha declines from 1.19% a month to 0.45% a month when factoring in liquidity availability.

The last column of Table 7 reports that the IVOL alpha disappears after controlling for liquidity in a slightly different way (see Ang et al. 2006). We first grouped returns into liquidity terciles and then IVOL quintiles within each liquidity tercile; we then computed average returns across the three DVOL groups. We used these average returns to estimate IVOL alphas for each IVOL quintile. As shown in Table 7, when calculated in this way, no anomalous returns are found—and alpha is the same for both high-IVOL and low-IVOL stocks and thus insignificant for the zero-cost long–short quintile portfolio.

Similarly, we earlier found a *negative* alpha for the zero-cost equal-weighted portfolio when using stocks priced above $5; that is, higher-risk stocks generate higher excess returns on an equal-weighted basis. More specifically, we found that, consistent with theory, equal-weighted excess portfolio returns monotonically increase with increases

Table 5. Abnormal Returns for IVOL-Based Portfolios: Monthly Regressions of IVOL Quintile Portfolio Returns, July 1963–December 2010 (*t*-statistics in parentheses)

Rank	Month $t + 1$ (1)	Average Price ($) (2)	Average NonZEROR (3)	Average Amihud (4)	Average DVOL ($ millions) (5)
1	0.08*	55.6	78.0%	0.37	256.7
	(1.87)				
2	0.07	29.2	82.7	0.75	178.9
	(1.13)				
3	0.09	19.2	82.2	1.60	121.4
	(1.09)				
4	–0.30**	13.5	80.7	3.99	79.8
	(–2.35)				
5	–1.11***	7.03	77.3	27.36	40.0
	(–5.90)				
1 – 5	1.19***				
	(6.04)				

Note: See notes to Table 2.

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

Table 6. Abnormal Returns for IVOL-Based Portfolios Excluding Penny Stocks: Monthly Regressions of IVOL Quintile Portfolio Returns, July 1963–December 2010 (*t*-statistics in parentheses)

Rank	Month $t + 1$ (no $P < \$5$) (1)	Average Price ($) (2)	Average NonZEROR (3)	Average Amihud (4)	Average DVOL ($ millions) (5)
1	0.12***	61.5	80.0%	0.33	272.1
	(2.64)				
2	0.07	35.6	84.2	0.57	214.0
	(1.26)				
3	0.07	24.5	84.5	0.88	165.3
	(1.06)				
4	0.03	19.5	84.8	1.30	135.2
	(0.27)				
5	–0.26*	14.3	85.4	2.58	111.1
	(–1.82)				
1 – 5	0.38**				
	(2.47)				

Note: Table 6 is similar to Table 5 but excludes penny stocks (i.e., stocks priced under $5 a share).

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

in portfolio risk quintiles when portfolios are adjusted for each of our various liquidity metrics.

The overall evidence presented in the previous tables suggests that the profitability of an IVOL-based trading strategy is meaningfully reduced, or even eliminated, in the subsample of stocks with high levels of liquidity. In other words, poor liquidity availability materially influences the ability of arbitrageurs to extract alpha when forming equal-weighted and value-weighted zero-cost portfolios.

The results reported in **Table 8** are very similar to those presented in Tables 3–7 except that

Table 7. IVOL-Based Portfolio Alpha and Liquidity: Monthly Regressions of IVOL Quintile Portfolios Grouped by High/Mid/Low Liquidity, July 1963–December 2010
(*t*-statistics in parentheses)

	Month $t + 1$ (no $P <$ \$5)	Low DVOL	Mid DVOL	High DVOL	Month $t + 1$ (average of DVOL)
Rank	(1)	(2)	(3)	(4)	(5)
1	0.12***	0.10	0.02	0.11**	0.08
	(2.64)	(1.42)	(0.29)	(2.34)	(1.53)
2	0.07	0.27***	0.04	0.08	0.13**
	(1.26)	(3.25)	(0.51)	(1.56)	(2.27)
3	0.07	0.41***	0.20**	0.12*	0.24***
	(1.06)	(4.83)	(2.23)	(1.82)	(3.96)
4	0.03	0.50***	0.13	0.05	0.23***
	(0.27)	(4.84)	(1.39)	(0.48)	(3.08)
5	–0.26*	0.60***	0.05	–0.34**	0.10
	(–1.82)	(4.66)	(0.37)	(–2.18)	(0.95)
1 – 5	0.38**	–0.50***	–0.02	0.45***	–0.02
	(2.47)	(–3.78)	(–0.19)	(2.71)	(–0.20)

Note: See note to Table 6.

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

Table 8. IVOL-Based Portfolio Alpha and Liquidity: Monthly Regressions of IVOL Quintile Portfolios Grouped by High/Mid/Low Amihud Liquidity Measure, July 1963–December 2010
(*t*-statistics in parentheses)

	Month $t + 1$ (no $P <$ \$5)	Low Amihud	Mid Amihud	High Amihud	Month $t + 1$ (average of Amihud)
Rank	(1)	(2)	(3)	(4)	(5)
1	0.12***	0.13***	0.02	0.24***	0.13**
	(2.64)	(2.59)	(0.33)	(2.87)	(2.36)
2	0.07	0.08	0.08	0.41***	0.19***
	(1.26)	(1.64)	(0.95)	(4.70)	(3.47)
3	0.07	0.08	0.17**	0.37***	0.21***
	(1.06)	(1.28)	(2.03)	(3.72)	(3.20)
4	0.03	0.08	0.17	0.45***	0.24***
	(0.27)	(1.15)	(1.63)	(4.12)	(3.20)
5	–0.26*	–0.20	–0.02	0.57***	0.11
	(–1.82)	(–1.37)	(–0.14)	(4.25)	(1.02)
1 – 5	0.38**	0.33**	0.04	–0.33**	0.02
	(2.47)	(2.07)	(0.27)	(–2.39)	(0.13)

Note: See note to Table 6.

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

liquidity is based on the Amihud (2002) measure in Table 8. The abnormal return to the IVOL-based trading strategy for the $t + 1$ period (columns 2 and 3 of Table 3), after controlling for liquidity, is substantially reduced for value-weighted portfolios. Altogether, our results suggest that investors will experience economically meaningful barriers (e.g., higher transaction costs) to arbitraging abnormal returns associated with the low-IVOL anomaly.

Discussion

In exploring the low-risk anomaly over the full study period (1963–2010), we first found that the various IVOL-based trading strategies we tested are profitable for value-weighted portfolios only with frequent rebalancing and not profitable at all, on average, when using BETA as the measure of risk. Holding periods beyond the first month produce little or no discernible alpha effect. We further found that for the monthly IVOL-based, rebalanced, value-weighted portfolios (where the potential alpha lies), the profitability of the zero-cost IVOL strategy is reduced by 60% or more in the subsample of stocks with relatively low transaction costs and high liquidity. This finding is in contrast to the findings of Ang et al. (2006, 2009), who suggested that transaction costs have little impact on alpha, although they did not consider the impact of penny stocks.

We also found that since 1990, the alpha associated with the low-risk effect for value-weighted portfolios has disappeared within the more liquid universe of nonpenny stocks. Specifically, over 1991–2010 (in unreported results), we found an insignificant zero-cost portfolio abnormal return of 0.24 (t-statistic = 0.80). Moreover, in exploring equal-weighted portfolios over all sample periods, we found that any alpha associated with the low-risk effect is eliminated in the subsample of stocks with ample liquidity for trading purposes.

The combined evidence from our liquidity measures consistently suggests meaningfully higher costs for arbitrageurs in attempting to exploit the low-risk anomaly. In short, our findings cast some doubt on the practical profitability of a low-risk trading strategy.

For all these reasons, our results lead us to conclude that the documented one-month, low-risk trading strategy continues to exist, to some extent, because of such significant barriers to arbitrage as transaction and liquidity costs, which constrain arbitrageurs from fully eliminating the anomalous returns.

Finally, our findings do not necessarily preclude the possibility of a profitable strategy based on the low-risk effect. Clearly, there are other potential approaches to capturing the low-volatility effect that may prove constructive in extracting abnormal returns. However, our study is comprehensive in its measures of risk and thus certainly raises important issues about implementing a successful low-risk arbitrage strategy for US equity portfolios. We look forward to more research on this intriguing topic.

Conclusion

Contrary to fundamental expectations, researchers have found that a strategy of buying prior low-volatility stocks and selling prior high-volatility stocks has historically generated substantial abnormal returns in the United States and international markets. Low-volatility effects are thus increasingly being used by portfolio managers in portfolio construction to extract excess returns. These results are particularly intriguing because, according to theory, higher expected return compensates for higher expected risk, not the other way around.

We showed that, in practice, the efficacy of exploiting the well-known low-volatility effect is more limited than widely believed. Our results indicate that the excess returns of zero-cost, low-risk portfolios (low risk minus high risk) reverse rather quickly, thereby requiring traders to rebalance frequently in any attempt to successfully extract profits. Furthermore, we found that the anomalous returns of value-weighted portfolios are largely eliminated when low-priced (less than $5) stocks are omitted—and are not at all present in equal-weighted portfolios. Moreover, any excess returns associated with the value-weighted, low-volatility effect are meaningfully reduced by high transaction costs beyond those directly associated with frequent rebalancing. Altogether, our evidence suggests that attempts to extract the alpha of zero-cost, low-volatility portfolios are substantially hampered by such market frictions as high transaction costs. Our results are based on a battery of empirical tests that separated the universe of stocks into high- and low-liquidity segments and then related the performance of those segments to the level of stock risk as adjusted for the well-known Fama–French factors of size and style. Traders thus face important limits to arbitrage that together have a significant negative impact on extracting expected abnormal returns associated with the low-volatility anomaly.

This article qualifies for 1 CE credit.

Notes

1. Recent research suggests that the negative relationship between idiosyncratic volatility and subsequent returns as reported in Ang et al. (2006, 2009) can be explained by their use of a short-term measurement of IVOL. For example, Fu (2009) and Huang, Liu, Rhee, and Zhang (2010) showed that the return association mostly results from the measurement approach of Ang et al. (2006, 2009) and that stocks with higher IVOL in one month have low returns the following month. In other words, the approach by Ang et al. essentially captures a large return-reversal effect. Also, Fu (2009) demonstrated that

the idiosyncratic volatility forecast from an EGARCH model is significantly positively related to subsequent returns. Finally, using various IVOL measures, Bali and Cakici (2008) found no significant relationship between IVOL and expected returns.

2. Our study differs from those that express the low-risk anomaly through risk parity portfolios (see Asness, Frazzini, and Pedersen 2012). Risk parity seeks to capture excess returns related to risk aversion by equalizing risk across asset classes and thus overweighting safer assets relative to the historical market-weighted portfolio.
3. We obtained the Fama–French factors ($R_m - R_f$, *SMB*, and *HML*) and the risk-free rate from Ken French's website (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french).
4. Lesmond et al. (1999) measured transaction costs as the proportion of zero-return days. As such, their measure is inversely related to liquidity (e.g., higher transaction costs mean a lower level of liquidity). For our liquidity metric, we simply used the inverse of their measure—that is, the proportion of *non*-zero-return days. A low value for our measure would thus correspond to a low level of liquidity.
5. In unreported results, we found that our IVOL spread portfolio also generates insignificant abnormal returns in each month from month $t + 4$ to month $t + 12$.
6. We found similar results when testing a five-year portfolio formation period (IVOL60).
7. The positive association between trading volume and stock returns for the most liquid stocks reported in Table 7 is consistent with the findings of Gervais, Kaniel, and Mingelgrin (2001).

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