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Low-Volatility Cycles: The Influence of Valuation and Momentum on Low-Volatility Portfolios

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Research showing that the lowest-risk stocks tend to outperform the highest-risk stocks over time has led to rapid growth in so-called low-risk equity investing in recent years. The authors examined the performance of both the low-risk strategy previously considered in the literature and a beta-neutral low-risk strategy that is more relevant in practice. They found that the historical performance of low-risk investing, like that of any quantitative investment strategy, is time varying. They also found that both low-risk strategies exhibit dynamic exposure to the well-known value, size, and momentum factors and appear to be influenced by the overall economic environment. Their results suggest that time variation in the performance of low-risk strategies is probably influenced by the approach to constructing the low-risk portfolio strategy and by the market environment and associated valuation premiums.

In what is often referred to as the low-volatility anomaly, researchers have shown that measures of prior stock price variability—including total return volatility, idiosyncratic volatility, and beta—relate to future performance but not necessarily in the way theory suggests: namely, that investors demand higher returns as compensation for higher expected risk. Instead, researchers have found, empirically, that the lowest-risk stocks tend to outperform the highest-risk stocks (Black 1972; Black, Jensen, and Scholes 1972). The finding of a negative risk–return trade-off contradicts the most basic principle of financial economics and, in so doing, suggests the possibility of a profitable trading strategy that buys low-volatility stocks and sells short high-volatility stocks. Consequently, a growing number of researchers have continued to examine the existence of and possible explanations for the anomaly (see, e.g., Ang, Hodrick, Xing, and Zhang 2006, 2009; Blitz and van Vliet 2007; Bali and Cakici 2008; Clarke, de Silva, and Thorley 2010, 2014; Baker, Bradley, and Wurgler 2011; Li, Sullivan, and Garcia-Feijóo 2014, forthcoming 2016; Frazzini, Asness, and Pedersen 2014; Frazzini and Pedersen 2014).

The low-volatility anomaly has had a dramatic impact on the theory and practice of investment management. In practice, investors have seen an explosion of strategies designed to "take advantage" of the outperformance of low-volatility stocks. In just the past few years, investors have allocated more than $10 billion to low-volatility mutual funds and exchange-traded funds, with strong growth also observed in institutional portfolio allocations.[1] Most of the existing low-volatility strategies, however, are seemingly constructed purely on the basis of risk, with little attention paid to the other characteristics known to affect portfolio performance. In this article, we analyze the time-varying nature of low-risk strategies to include quantifying their relationship to other characteristics, such as size, value, and momentum, as well as the possible impact of the macroeconomic environment.

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The low-volatility strategies discussed in the literature typically approach low-volatility investing with models designed to separate out the high-volatility stocks from the low-volatility stocks via the creation of "risk quintiles." These strategies define risk as either (1) idiosyncratic volatility, using a multifactor model that controls for the well-known size (large versus small) and style (growth versus value) characteristics, or (2) beta from the CAPM. In our study, following Baker et al. (2011), we focused on beta as defined by the coefficient on the market factor from the CAPM. Specifically, we explored two low-risk strategies based on the CAPM beta and their dynamic performance.

In the low-volatility literature, the discussion of how low-volatility portfolios interact with the well-known characteristics of value, size, and momentum (Fama and French 1992, 1993; Jegadeesh and Titman 1993, 2001; Asness 1997) remains largely absent. We suggest that omitting these characteristics identified in the literature—and now widely understood to influence excess returns over time—is an important oversight. Our study extends the literature by exploring the importance of accounting for how low-risk portfolios evolve dynamically over time and how exposure to size, style, and momentum contributes to that evolution.

Using the CAPM beta as our measure of risk, we first updated earlier findings through 2012, confirming previously reported evidence of the existence of the low-risk anomaly in a sample of US large-cap stock returns.[2] We then calculated the time-varying valuation (using book-to-price ratios) of low-volatility portfolios, demonstrating how this valuation at any point in time relates to the future performance of low-risk portfolios in our sample. We documented the presence of secular environments in which, consistent with theory, high-risk stocks tend to outperform low-risk stocks for extended periods. We then expanded this analysis by demonstrating that low-risk portfolio performance is sensitive to value and momentum over time, which suggests the importance of understanding how valuation and price momentum act as drivers of low-risk stock portfolios.

Data

We obtained stock return data from the Center for Research in Security Prices (CRSP) for all stocks trading on the NYSE, the American Stock Exchange (AMEX), and NASDAQ for 1925–2012. However, we focused our analysis primarily on 1968–2012, following the existing literature. (Our review of the evidence for the longer period appears near the end of this article.) We focused on common stocks only (share codes 10 and 11). For delisted companies, we fetched the returns in the delisting month from the CRSP. If the delisting was for performance-related reasons, however, we set the delisting return equal to –55% if trading on NASDAQ or to –30% if trading on the NYSE or AMEX (for an analysis of CRSP delisting bias, see Shumway 1997; Shumway and Warther 1999). To minimize the impact of illiquid stocks, we excluded stocks with prices lower than \$2 or greater than \$1,000. Following Baker et al. (2011), we focused on large-cap stocks, defined as those companies in the top third (based on market capitalization using NYSE breakpoints). We focused on large-cap stocks because implementing a low-risk strategy may require frequent rebalancing (Li et al. 2014) and thus the efficacy of implementation within the small-cap stock universe is questionable. (Consistent with this previous research, we found, and confirmed empirically, that outperformance of low-risk stocks in the large-cap universe is diminished versus their small-cap counterparts over the sample period.)

We obtained our accounting data from Compustat. We used information reported as of December of each fiscal year (i.e., we used only companies with a December fiscal year-end), following Fama and French (1992) in the computation of book equity. To minimize the impact of newly listed companies, we required two years of data for a company to be included in the sample. For years prior to 1951, we obtained data from Ken French's website.[3]

Model and Analysis

As mentioned earlier, we focused on large-cap stocks only, using the top third of companies (based on NYSE breakpoints). For each month, we sorted stocks into quintiles on the basis of risk, where risk is measured by beta (b_i) as estimated from the CAPM (market model), following Baker et al. (2011):

$$R_{i,t} - R_{f,t} = a_i + b_i\left(R_{M,t} - R_{f,t}\right) + \varepsilon_{i,t}. \qquad (1)$$

We estimated beta monthly in regressions of excess returns on the CRSP value-weighted index over the previous 60 months (a minimum of 24 months). Specifically, for each month t, we estimated beta for each stock using the prior 60 months of returns, including month t. We sorted stocks into risk quintiles on the basis of the estimated beta and computed value-weighted average returns for each quintile in month $t + 1$. The average of the monthly value-weighted stock returns by risk quintile is reported in **Table 1**. We calculated the arithmetic and geometric average returns for the quintile portfolios over the sample period.

Column 2 of Table 1 reports the value-weighted average monthly raw returns (i.e., not excess returns) on risk quintile portfolios with beta as the measure of risk. Consistent with previous literature, we found that stocks in the lowest-beta quintile outperform those in the highest-beta quintile over the sample period for both arithmetic average and geometric average returns. For example, the average arithmetic return on the lowest-beta quintile is 0.89% over 1968–2012, whereas the average arithmetic return on the highest-beta quintile is 0.73%.

Although the average betas rise meaningfully for each successively higher quintile, average returns for the first four beta quintiles are all roughly equivalent for both arithmetic and geometric means. The average returns for the highest-beta quintile then decline, with monthly arithmetic means falling from roughly 0.90% in the lower quintiles to 0.73% in the highest quintile and monthly geometric returns similarly falling from about 0.80% to 0.47%. Importantly for investors, as Table 1 shows, the corresponding Sharpe ratios decline for each successively higher quintile, with Sharpe ratios falling meaningfully from 0.45 in the lowest quintile to 0.15 in the highest quintile. Thus, a low-volatility portfolio has historically offered a compelling risk-adjusted return three times higher than that of a high-risk portfolio.

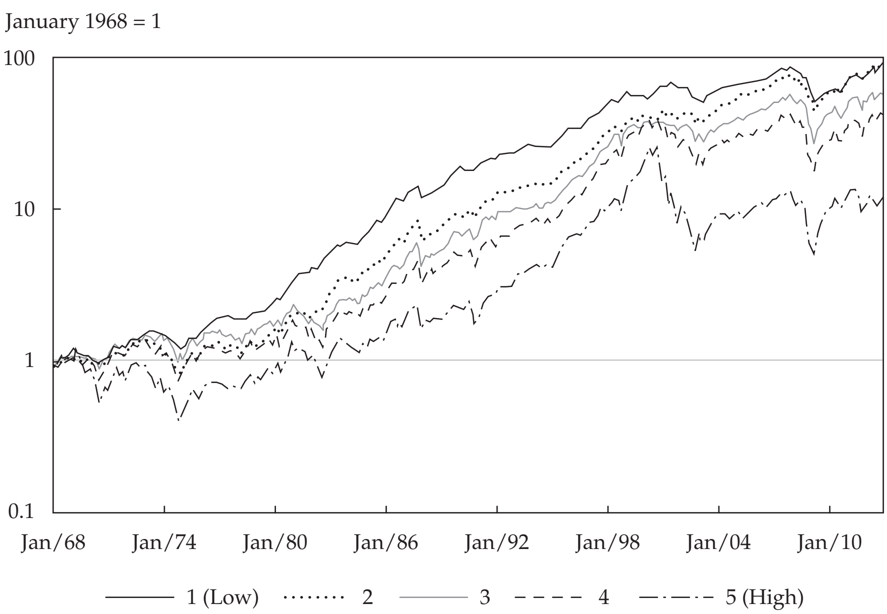
Figure 1 depicts geometric returns of risk quintile portfolios by plotting the cumulative growth of $1 invested in each quintile portfolio over the sample period. Consistent with the findings of Baker et al.

Table 1. Risk and Return Characteristics of Beta Portfolios, 1968–2012 (*t*-statistics in parentheses)

Beta Quintile	Value-Weighted Return (arithmetic mean)	Value-Weighted Return (geometric mean)	Beta	Sharpe Ratio
1 (low)	0.89%	0.83%	0.47	0.45
2	0.92	0.84	0.76	0.42
3	0.86	0.75	0.98	0.32
4	0.85	0.70	1.24	0.27
5 (high)	0.73	0.47	1.76	0.15
Low – high	0.16	–0.02	–1.29	0.10
	(0.63)			
Beta neutral	0.71***	0.67	0.03	0.46
	(5.49)			
Market	0.87***	0.76	1.00	0.33
	(4.35)			

Notes: Table 1 reports raw return and risk characteristics by beta quintile over 1968–2012 (540 months). The sample includes large-cap stocks only (largest one-third using NYSE breakpoints). Arithmetic mean is the average value-weighted monthly raw return. Beta is computed monthly in regressions of excess returns on the CRSP value-weighted index over the previous 60 months.
***Significant at the 1% level.

Figure 1. Cumulative Returns on Beta Quintile Portfolios, January 1968–December 2012



Note: Beta quintile portfolios are derived from the universe of large-cap stocks.

(2011), geometric returns of high-beta stocks underperform those of low-beta stocks, with the four lowest quintiles performing somewhat closely together versus the highest quintile.[4]

The pertinent literature typically examines the performance of a zero-cost strategy that goes long the stocks in one of the extreme quintiles and short the stocks in the opposite extreme quintile (e.g., Ang, Hodrick, Xing, and Zhang 2009). As reported in Table 1, the raw return on the zero-cost spread portfolio that goes long the lowest-risk quintile and short the highest-risk quintile is 0.16%, which is insignificantly different from zero at the 5% level (the *t*-statistic is 0.63).[5] The Sharpe ratio is 0.10.[6] This finding suggests that any outperformance of a risk-based zero-cost spread portfolio is not economically meaningful over time, on average, for the large-cap universe of stocks.[7] This finding is consistent with Clarke, de Silva, and Thorley (2014), who used a VMS (volatile minus stable) factor.

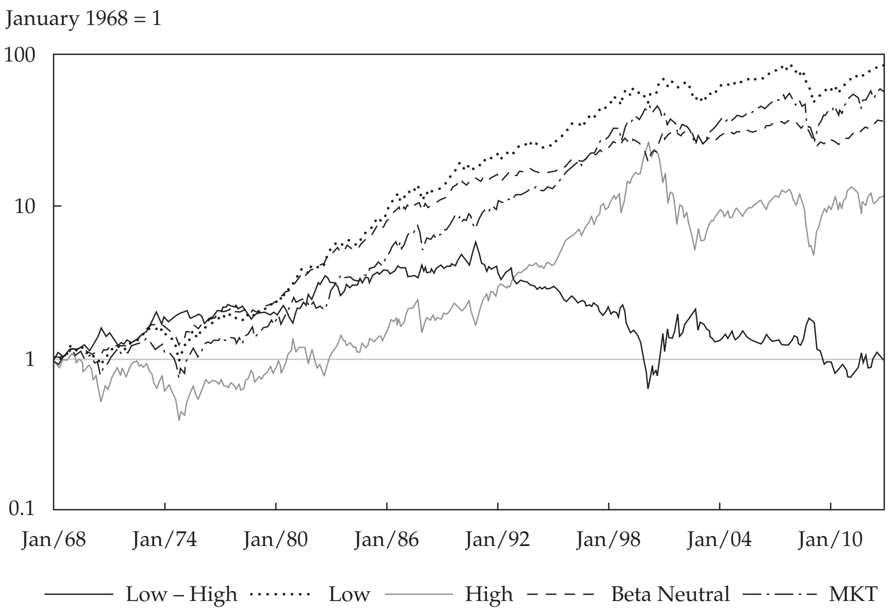
In **Figure 2**, we can see that the zero-cost long–short portfolio generates a cumulative return of roughly zero over our sample period, thus underperforming the market. Therefore, a zero-cost arbitrage portfolio that is long low-risk stocks and short high-risk stocks performs relatively poorly, on average, over time (Li et al. 2014).

However, a potential concern with the zero-cost long–short portfolio, which equally dollar-weights the long side and the short side, is that it results, in our case, in a negative portfolio beta over the sample period—an undesirable trait for most investors, who do not ordinarily seek to have a persistently net short position with respect to the equity market over time. In response, we can consider an alternative approach to constructing a long–short low-risk portfolio—one that is roughly beta neutral (a beta of zero), on average, over time (Frazzini and Pedersen 2014).[8] In contrast to our zero-cost long–short portfolio, which is 100% long the low-risk quintile and 100% short the high-risk quintile, our beta-neutral portfolio is formed by going long 100% of the lowest-risk stocks while shorting only 25% of the highest-risk quintile over the sample period. Of course, such a beta-neutral portfolio would possess far less (if any) market risk over time and would thus offer a more practical approach to capturing the observed risk-related anomalous returns.

To compare the two long–short portfolio betas from Table 1, we calculate that our zero-cost long–short portfolio has an average market-related beta of –1.29 ($1.0 \times 0.47 - 1.0 \times 1.76 = -1.29$), whereas our beta-neutral portfolio has an average market beta of about zero ($1.0 \times 0.47 - 0.25 \times 1.76 = 0.03$). From Figure 2, we see that our beta-neutral long–short portfolio meaningfully improves performance versus our zero-cost long–short portfolio over the study period. That the beta-neutral portfolio underperforms the market portfolio over the study period is not a surprise, given that it has a zero beta.

Table 1 shows a highly statistically significant monthly average arithmetic return of 0.71% for the beta-neutral portfolio over the sample period, versus the statistically insignificant average arithmetic

Figure 2. Cumulative Returns on Select Beta-Focused Portfolios and the Market, January 1968–December 2012



Note: Beta quintile portfolios are derived from the universe of large-cap stocks, and "the market" represents all stocks.

return of 0.16% for the zero-cost long–short portfolio. The resulting Sharpe ratio of 0.46 for the beta-neutral long–short portfolio is also much improved over the Sharpe ratio of 0.10 for the zero-cost long–short portfolio and is roughly equivalent to the Sharpe ratio for the lowest-risk quintile. For comparison, the average arithmetic return on the market (i.e., the CRSP value-weighted index) is 0.87%, with a Sharpe ratio of 0.33.

To summarize, we observe little relationship between risk and return for the four lower-risk quintiles but do observe a marked underperformance for the highest-risk quintile versus the lower-risk quintiles. Sharpe ratios decline meaningfully as portfolio risk rises, leading to declining returns per unit of risk. In an attempt to arbitrage the lower returns of the highest-risk quintile relative to those of the lowest-risk quintile, we observe no outperformance for a zero-cost long–short portfolio with respect to both arithmetic and geometric returns. However, our alternative long–short beta-neutral portfolio demonstrates much stronger performance over time, with a Sharpe ratio in line with that of the lowest-risk quintile and higher than that of the market portfolio over the sample period.

These results, however, conceal secular environments in which high-risk stocks tend to *outperform* low-risk stocks for extended periods, consistent with theory. Our evidence suggests that these periods of outperformance by high-risk stocks correspond to initial relative undervaluation of those stocks. At this point in our study, we turned our attention to exploring this time-varying performance in more detail.

Low Risk and Valuation

We examined the relative valuation levels of the lowest- and highest-risk quintiles over time, using the average book-to-price ratio (B/P) for each of the two portfolios for each month. We defined our chosen valuation measure, B/P, as the book value of common equity divided by the market value of equity. We computed book equity as in Fama and French (1992), using information from the prior fiscal year. We computed market capitalization at the end of each month (Asness and Frazzini 2013) as share price times the number of shares outstanding.

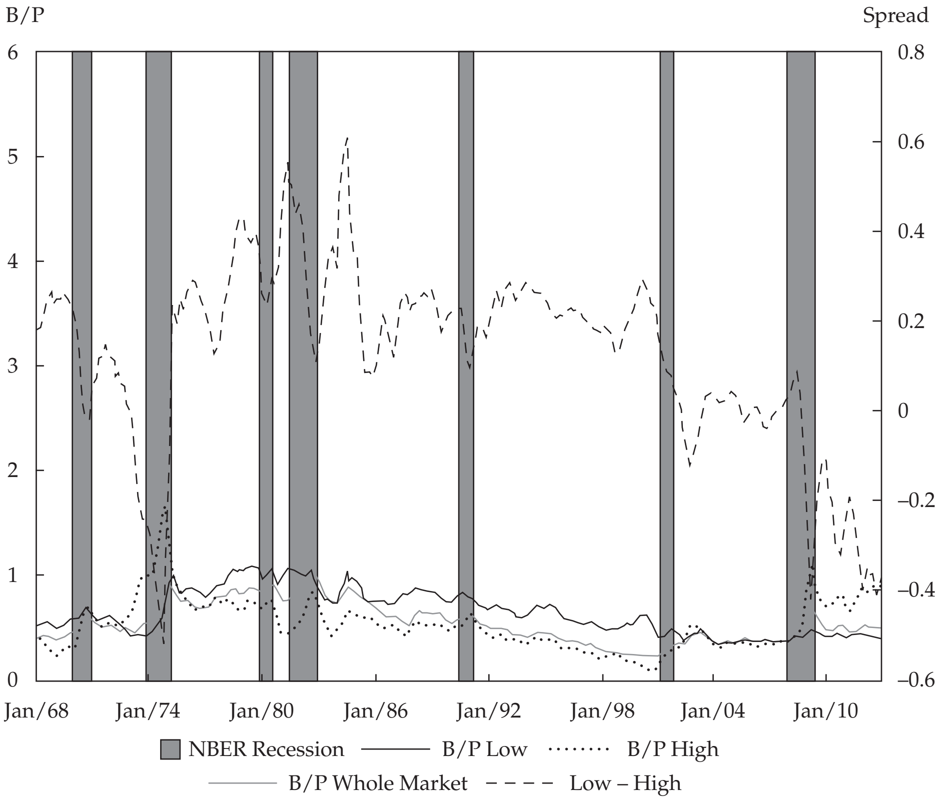
Specifically, as discussed earlier, for each month t, we estimated beta for each stock, using the prior 60 months of returns. We then sorted stocks into risk quintiles on the basis of the estimated beta and computed value-weighted average returns for each quintile in month $t + 1$. For valuation purposes, we then calculated the B/P for each stock for each month t, which allowed us to compute the mean B/P for the portfolio of stocks in the lowest-risk quintile and for the portfolio of stocks in the highest-risk quintile for each month. We also subtracted the average B/P of the high-beta quintile from the average B/P of the low-beta quintile, denoting the difference as "B/P spread." We had one value of B/P per month for each risk quintile and for the B/P spread over the sample period.

We began our exploration of the interaction of valuation and momentum in low-risk portfolios with a visual inspection. **Figure 3** plots the time-varying mean level of B/P for both the market portfolio and the large-cap universe, the B/P for stocks in the lowest- and highest-beta quintiles, and the B/P spread. The shaded areas correspond to National Bureau of Economic Research (NBER) recession periods. Two observations stand out. First, high-beta B/Ps tend to "spike" during recessions, compressing the spread between low- and high-beta B/Ps—sometimes to the point of reversing the typical relationship. That is, B/Ps for high-risk stocks are typically lower than B/Ps for low-risk stocks (i.e., the B/P spread is positive), indicating a valuation premium for high-risk stocks. However, during recessionary periods, the prices of high-beta stocks decline significantly and the B/P spread falls, perhaps owing to a "flight to safety." Occasionally during such periods, the valuation spread even becomes negative, so high-beta stocks trade at a relative discount to low-beta stocks with respect to B/P.

Second, following recessions, the B/P spread tends to increase dramatically, perhaps because investors increase their risk appetites and the valuation of high-risk stocks reverts to more normal levels. Note that the spread moved to a negative value during the 2008 global financial crisis but had not reverted to more normal levels by the end of our study period, 2012. Instead, after initially bouncing from a 30-year low following the 1974–75 recession, the B/P spread moved back down to its 2008 crisis lows, indicating a continued deep relative valuation discount for high-risk stocks. Only time will tell whether the B/P spread will revert to more normal levels.

We then turned our attention to a more formal empirical analysis of the relationship between low-risk portfolio performance and the well-known value and momentum factors. We first examined the relative performance in month $t + 1$ of low-risk and high-risk stocks in accordance with varying initial (time t) levels of B/P for those stocks. To do so, we used the approach described earlier whereby we formed risk quintile portfolios and then calculated the B/P spread for each month by subtracting the average B/P of the high-beta quintile from the average B/P of the low-beta quintile. We then created B/P spread quintiles by rank-ordering the monthly B/P spread (the data cover

Figure 3. Evolution of Average B/P for Select Beta-Focused Portfolios and the Market, January 1968–December 2012



Note: Beta quintile portfolios are derived from the universe of large-cap stocks, and "the market" represents all stocks.

1968–2012). We deleted the top and bottom 0.5% of the B/P observations to minimize the impact of extreme observations.

Panel A of **Table 2** reports statistics similar to those in Table 1 but separates the long–short risk spread portfolio into B/P spread quintiles. Importantly, this approach allowed us to identify months in which the B/P spread was low and months in which the spread was high for the long–short risk spread portfolio. We were thus able to examine the future excess return performance of the zero-cost and beta-neutral spread portfolios relative to their initial valuations as based on the B/P spread quintiles.

Panel A of Table 2 reports value-weighted average returns in month $t + 1$ on beta quintile portfolios separated according to B/P spread in month t.[9] Our stock portfolios are rebalanced monthly in accordance with the risk quintiles discussed earlier.[10] Focusing first on the highest B/P spread portfolios (when low-beta stocks begin in month t at a valuation discount relative to high-beta stocks), we can see that low-beta stocks meaningfully outperform high-beta stocks, on average, in the next month over the sample period. More specifically, as shown in the bottom row of Panel A (when low-beta stocks are at the deepest valuation discount relative to high-beta stocks), the average one-month-ahead return on the beta quintiles ranges from 1.42% for the lowest-risk quintile to 0.55% for the highest-risk quintile. For this B/P spread quintile, the zero-cost spread portfolio average return is a positive, though statistically insignificant (the t-statistic is 1.46), 0.87%. However, as can be seen from the first row of Panel A, when the B/P spread is most negative—that is, when initial valuation levels in month t reflect the greatest relative valuation discount for high-risk stocks—the high-beta quintile tends to outperform the low-beta quintile in the next month. The average return is 0.13% for the lowest-risk quintile and 0.35% for the highest-risk quintile. The average return on the zero-cost spread portfolio over the sample period is –0.21%, which is insignificantly different from zero (the t-statistic is –0.35). The results for our beta-neutral portfolio are similar, if slightly improved, to those of our zero-cost long–short portfolio. None of the zero-cost long–short portfolio average returns are statistically different from zero, whereas the beta-neutral portfolio returns are significant on three occasions (in the two quintiles with the highest B/P spreads and when low-beta stocks have low relative initial valuations).

Panel B of Table 2 extends the evidence from Panel A by reporting annualized excess returns of our two long–short spread portfolios over longer future

Table 2. Returns on Beta Risk Portfolios by B/P Spread, 1968–2012 (*t*-statistics in parentheses)

B/P Spread Quintiles (B/P for low-beta stocks – B/P for high-beta stocks)				Average Value-Weighted Return in Month $t+1$						
Quintile	From	To	Avg. Monthly No. Stocks	1 (low beta)	2	3	4	5 (high beta)	Low-Beta – High-Beta Avg. Ret.	Beta-Neutral Avg. Ret.
Panel A										
1 (low)	–0.55	–0.01	104	0.13%	0.50%	0.41%	0.39%	0.35%	–0.21%	–0.12%
									(–0.35)	(–0.42)
2	–0.01	0.12	101	1.22	1.40	1.31	1.34	1.01	0.21	0.69**
									(0.37)	(2.83)
3	0.12	0.22	108	0.90	0.96	1.28	1.38	1.30	–0.40	0.21
									(–0.63)	(0.66)
4	0.22	0.27	106	1.01	1.08	0.94	0.87	0.77	0.23	0.47*
									(0.51)	(1.79)
5 (high)	0.27	0.59	94	1.42	0.99	0.71	0.65	0.55	0.87	0.80**
									(1.46)	(2.50)

			Annualized Future Cumulative Excess Return on Low-Beta – High-Beta Strategy				
Panel B			$t+12$	$t+24$	$t+36$	$t+48$	$t+60$
1 (low)	–0.55	–0.01	–5.80%	–3.63%	–3.37%	–3.29%	–2.83%
2	–0.01	0.12	4.64	–0.67	0.24	0.33	0.24
3	0.12	0.22	–6.84	–5.30	–3.99	–2.50	–2.23
4	0.22	0.27	3.36	1.62	–2.45	–3.11	–2.81
5 (high)	0.27	0.59	12.52	8.45	6.45	4.96	4.40

			Annualized Future Cumulative Excess Return on Beta-Neutral Strategy				
Panel C			$t+12$	$t+24$	$t+36$	$t+48$	$t+60$
1 (low)	–0.55	–0.01	2.30%	3.71%	3.06%	1.04%	1.62%
2	–0.01	0.12	3.97	1.51	2.06	2.47	2.61
3	0.12	0.22	2.53	2.28	3.31	3.74	3.49
4	0.22	0.27	5.63	4.73	4.09	4.35	4.34
5 (high)	0.27	0.59	10.27	10.42	9.25	8.99	9.10

Notes: The sample includes a total of 540 months, or 540 observations of the B/P spread, so each row is based on 108 months. "Avg. Monthly No. Stocks" is the average number of stocks in the long top quintile and short bottom quintile portfolios (the number of stocks in the long and short portfolios is the same).

*Significant at the 10% level.

**Significant at the 5% level.

($t+n$) investment horizons, rebalanced monthly. The main difference is that these results have been extended from a one-month ($t+1$) horizon to include cumulative returns for various future investment horizons up to five years ($t+60$). Note that because these results are reported for overlapping periods, the usual caveats apply in interpreting them. We again observe that there is an interaction between the performance of rebalanced low-risk minus high-risk portfolios and initial valuation levels. That is, a portfolio of high-risk stocks outperforms a portfolio of low-risk stocks from one year ($t+12$) to five years ($t+60$) following periods that begin with a negative B/P spread generating negative returns for the zero-cost, low-risk minus high-risk portfolio. Specifically, average annual cumulative returns are –5.80% in $t+12$ and around –2.83% for five years ($t+60$) following the initial valuation month, t. As mentioned before, a negative B/P spread (computed as low beta minus high beta) suggests that high-beta stocks are selling at a discount at the time of portfolio formation in initial month *t*. In contrast, low-risk stocks tend to outperform when the B/P spread is high at the time of portfolio formation or when low-risk stocks begin the period at a relative discount, as shown in the bottom row of Panel B.

Panel C of Table 2 shows that the beta-neutral portfolio improves overall portfolio performance over each of the investment horizon periods versus the regular long–short portfolio, which has a negative market beta on average. Consistent with expectations, the beta-neutral portfolio returns generally increase with improved valuation of the low-risk stocks at the time of portfolio formation. Interestingly, unlike the zero-cost risk portfolio, the beta-neutral portfolio experiences no negative returns for any holding period, no matter the starting valuation.

Although these findings are new, the overall evidence from Table 2 should come as no surprise—after all, finance theory strongly supports the notion that valuation matters. As demonstrated by the zero-cost, low-risk minus high-risk portfolio returns, the lowest-risk portfolio outperforms the highest-risk portfolio in future periods only when valuation is strongly in its favor. Specifically, only when low-risk stocks are initially at their most attractive valuation levels relative to high-risk stocks (shown by the quintile with the highest B/P spread in the bottom row of Panel B in Table 2) do low-risk stocks consistently outperform high-risk stocks across all portfolio investment horizons. The remaining valuation quintiles report a mix of positive and negative spread returns for the zero-cost, low-risk minus high-risk spread portfolios across the various investment horizons. The practical implication of our results is the possibility that the performance of low-risk strategies is influenced by time-varying valuation premiums. To be clear, we are not suggesting that any timing of these premiums should be attempted or could even be accomplished.

These results suggest that risk-based portfolios may have important time variation in their exposure to well-known characteristics that influence stock returns. We took this idea and built on the evidence from Table 2 with a more formal analysis. **Table 3** reports monthly alpha results and *t*-statistics with respect to the intercept from regressions of our two low minus high spread portfolio excess returns on the market in Panel A and on the well-known four factors of Fama and French (1993) and Jegadeesh and Titman (1993)—the market, SMB (small minus big), HML (high minus low), and MOM (momentum)—in Panel B. As noted by, for example, Novy-Marx (2012), the regression intercept's *t*-statistic is the information ratio of the low-risk minus high-risk strategy benchmarked to the strategies embodied in the explanatory variables (i.e., the market in Panel A and size, value, and momentum in Panel B). A statistically significant intercept indicates that inclusion of the low-risk strategy can result in an information ratio improvement over one that can be achieved by using the strategies embodied in the explanatory variables alone. In contrast, an insignificant intercept indicates that there may be no benefit to the portfolio strategy in terms of information ratio improvement in adding the low-risk strategy versus a strategy using the market, size, value, and momentum alone. We emphasize, however, that we do not seek to "enhance" existing low-risk strategies. Rather, our aim is to better understand the dynamic interactions between low-risk portfolios and such characteristics as style and momentum.[11]

As before, we estimated beta for each stock using the prior 60 months of returns and sorted stocks into risk quintiles on the basis of their estimated beta. We rebalanced portfolios for each month *t* and computed average returns for each month $t + 1$. We then ran regressions of excess returns on the low-volatility strategy against systematic risk factors using overlapping sets of 12 months, 24 months, 36 months, and so on. We report the average of the estimated alphas (Table 3) and slope coefficients (**Table 4**). For example, for the $t + 12$ results shown in column 4 of Panel A in Table 3, we ran regressions of the beta spread quintile excess returns on the market risk premium (MKT) using the months of January 1968–December 1968, February 1968–January 1969, March 1968–February 1969, and so on (when December 1967, January 1968, and February 1968, respectively, are in the same B/P quintile). Given our use of overlapping periods, in reporting *t*-statistics we have attempted to correct for autocorrelation and heteroskedasticity (see Newey and West 1987), although the reliability of the *t*-statistics is nonetheless probably still weakened.

Panel A of Table 3 shows that unconditional alphas for both types of our long–short portfolios tend to be insignificantly different from zero when the B/P spread is within the lowest three quintiles (i.e., when low-risk stocks tend to be at a valuation premium to high-risk stocks). However, when the B/P spread is within its two highest valuation quintiles (i.e., when high-risk stocks are trading at a premium), alphas tend to be significantly positive for our two long–short portfolios for up to five years after portfolio formation. Furthermore, alphas in the two quintiles with the highest B/P spreads are significantly positive, as shown by the corresponding *t*-statistics.

Our investigation of the connection between strategies based on style and momentum and low-volatility strategies is reported in Panel B of Table 3, which repeats the regression analysis from Panel A but conditionally includes the four factors of MKT, HML, SMB, and MOM. Comparing Panels A and B, when the B/P spread is within its two highest valuation quintiles (i.e., when high-risk stocks are trading at a premium), we observe a decline in the conditional alphas and *t*-statistics for the low-volatility strategy once the additional factors are included in the regressions. That is, the mostly significant alphas in the bottom two rows of Panel A are diminished and only occasionally significant for both of our long–short portfolio strategies. This finding suggests an important association between the valuation and performance of portfolios formed on beta and strategies based on value and momentum (recall that the smallest stocks are excluded from the sample).

Table 3. Risk-Adjusted Returns on Beta Spread Portfolios by B/P Spread, 1968–2012 (*t*-statistics in parentheses)

B/P Spread Quintile (B/P for low-beta stocks – B/P for high-beta stocks)			Estimated Alphas of Monthly Regressions of Low-Beta – High-Beta Strategy against Market				
Quintile	From	To	*t* + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
Panel A							
1 (low)	–0.55	–0.01	0.07%	0.38%	0.30%	0.08%	0.14%
			(0.10)	(0.50)	(0.73)	(0.43)	(1.15)
2	–0.01	0.12	0.45	0.21	0.37	0.37	0.36
			(0.90)	(0.26)	(1.02)	(1.53)	(1.23)
3	0.12	0.22	–0.11	0.03	0.23	0.33*	0.37**
			(–0.34)	(0.11)	(1.04)	(1.73)	(2.25)
4	0.22	0.27	0.46***	0.35**	0.25	0.32*	0.39***
			(2.62)	(2.30)	(1.64)	(1.68)	(2.69)
5 (high)	0.27	0.59	1.04***	1.02***	0.90**	0.92***	0.92***
			(5.11)	(3.76)	(2.02)	(3.11)	(4.28)
			Estimated Alphas of Monthly Regressions of Beta-Neutral Strategy against Market				
			t + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
1 (low)	–0.55	–0.01	0.00%	0.16%	0.12%	–0.01%	0.02%
			(–0.01)	(0.38)	(0.42)	(0.12)	(0.18)
2	–0.01	0.12	0.15	0.06	0.15	0.15	0.15
			(0.44)	(0.13)	(0.68)	(0.89)	(0.67)
3	0.12	0.22	–0.09	0.03	0.14	0.20*	0.21**
			(–0.51)	(0.20)	(1.21)	(1.74)	(2.07)
4	0.22	0.27	0.27**	0.16**	0.10	0.17*	0.23***
			(2.56)	(1.98)	(1.11)	(1.68)	(2.91)
5 (high)	0.27	0.59	0.68***	0.63***	0.54**	0.52***	0.52***
			(5.09)	(4.47)	(2.47)	(3.29)	(4.07)
			Four-Factor Model–Estimated Alphas of Regressions of Low-Beta – High-Beta Strategy				
Panel B			*t* + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
1 (low)	–0.55	–0.01	0.10%	0.36%	0.37%	0.18%	0.22%
			(0.31)	(1.48)	(1.22)	(0.71)	(1.30)
2	–0.01	0.12	0.18	–0.04	0.04	0.15	0.15
			(0.66)	(–0.18)	(0.30)	(1.04)	(0.72)
3	0.12	0.22	0.03	–0.07	–0.06	–0.10	–0.11
			(0.16)	(–0.50)	(–0.38)	(–0.42)	(–0.67)
4	0.22	0.27	0.06	–0.12	–0.10	–0.03	–0.09
			(0.24)	(–0.69)	(–0.62)	(–0.20)	(–0.58)
5 (high)	0.27	0.59	0.40*	0.40**	0.27	0.30	0.29*
			(1.86)	(2.19)	(1.06)	(1.28)	(1.72)

(continued)

Table 3. Risk-Adjusted Returns on Beta Spread Portfolios by B/P Spread, 1968–2012 (*t*-statistics in parentheses) (Panel B continued)

B/P Spread Quintile (B/P for low-beta stocks – B/P for high-beta stocks)			Four-Factor Model–Estimated Alphas of Regressions of Beta-Neutral Strategy				
Quintile	From	To	*t* + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
1 (low)	–0.55	–0.01	0.03%	0.18%	0.19%	0.10%	0.11%
			(0.19)	(1.35)	(1.00)	(0.58)	(0.91)
2	–0.01	0.12	0.04	–0.02	0.01	0.06	0.06
			(0.20)	(–0.17)	(0.13)	(0.56)	(0.39)
3	0.12	0.22	–0.01	–0.03	–0.02	–0.05	–0.07
			(–0.07)	(–0.28)	(–0.20)	(–0.37)	(–0.64)
4	0.22	0.27	0.06	–0.14	–0.11	–0.04	–0.06
			(0.29)	(–1.13)	(–0.95)	(–0.34)	(–0.53)
5 (high)	0.27	0.59	0.34*	0.31**	0.18	0.17	0.16
			(1.85)	(2.24)	(0.86)	(0.80)	(1.02)

*Significant at the 10% level.
**Significant at the 5% level.
***Significant at the 1% level.

Table 4. Estimated Slope Coefficients of Beta Portfolios on HML and MOM Factors, 1968–2012

B/P Spread Quintile (B/P for low-beta stocks – B/P for high-beta stocks)			Average Beta on HML				
Quintile	From	To	*t* + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
Panel A							
1 (low)	–0.55	–0.01	0.11	0.16	0.24	0.33	0.31
2	–0.01	0.12	0.33	0.50	0.57	0.52	0.52
3	0.12	0.22	0.73	0.78	0.73	0.79	0.79
4	0.22	0.27	0.65	0.66	0.68	0.66	0.71
5 (high)	0.27	0.59	0.53	0.65	0.68	0.69	0.65
			Average Beta on MOM				
Panel B			*t* + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
1 (low)	–0.55	–0.01	0.18	0.08	0.10	0.08	0.03
2	–0.01	0.12	0.08	0.05	0.05	0.08	0.14
3	0.12	0.22	0.15	0.10	0.14	0.18	0.17
4	0.22	0.27	0.23	0.26	0.17	0.12	0.12
5 (high)	0.27	0.59	0.24	0.35	0.32	0.23	0.19

Importantly, the lower alphas and *t*-statistics indicate that portfolio performance and information ratios tend to degrade once value and momentum strategies are included along with the low-risk strategy. This finding is important for practitioners in implementing low-risk strategies. It indicates a historical interaction among low risk, value, and momentum that influences portfolio performance. Results for the beta-neutral strategy, also reported in Table 3, confirm the evidence from the original low-beta strategy.

Taken together, the results from Panels A and B in Table 3 reinforce and extend the evidence from Table 2. First, future performance of a low-risk trading strategy depends importantly on the initial price paid. In other words, extracting excess returns from the low-volatility anomaly depends on initial valuation levels: low-risk stocks tend to outperform over future investment horizons but only when initial valuations are in their favor. This finding highlights the connection between low-risk and value-based strategies. Moreover, once the well-known cross-sectional factors of style and momentum are controlled for, alphas and information ratios for our two long–short portfolio strategies decline. This finding indicates that there are important interactions among strategies based on these factors.

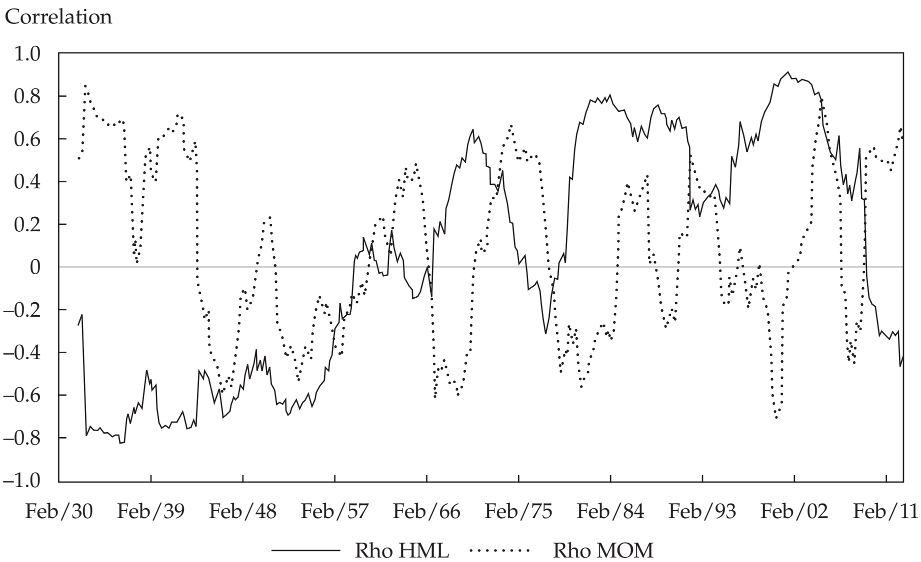
Interaction of Low Risk, Value, and Momentum

We then extended our analysis by further exploring the dynamic interactions of our two long–short portfolio strategies with value and momentum. Table 4 looks at the role of the value and momentum factors in explaining the risk exposure and performance of the low-risk minus high-risk spread portfolio over time. We applied the same estimation procedure that we used for Table 3, but Table 4 reports average exposures (i.e., slopes) for the HML and MOM factors over various forward-looking investment horizons. Panel A of Table 4 shows that beta spread portfolio returns tend to exhibit low levels of exposure to HML when the initial B/P spread is negative (i.e., when low-risk stocks begin the period at a valuation premium) but exposures to HML rise when the initial B/P spread is positive (i.e., when high-risk stocks trade at a premium). This finding is not surprising, because B/P and HML are directly connected in that B/P is the company characteristic that is used to determine the level of the HML factor and low-risk stocks outperform when their B/P is higher relative to their high-risk counterparts (quintiles with the highest B/P spreads).

Of greater interest is the evidence on momentum in Panel B of Table 4. Following portfolio formation, the beta spread strategy tends to exhibit higher exposure to MOM when the B/P spread begins the period in positive territory (i.e., when initial valuation levels favor low-risk stocks). This finding suggests that the performance of low-beta strategies is also influenced by the momentum factor. The combination of initial valuation levels and momentum sheds new light on the performance of low-risk strategies.

To aid understanding of our findings regarding the importance of style and momentum to low-risk portfolio performance, **Figure 4** plots the rolling four-year correlations between the return to the risk spread portfolio and the value (HML) and momentum (MOM) factors, all calculated as described earlier. Consistent with our earlier findings, the risk spread portfolio has historically shown meaningful cyclical exposure to both a value factor and a momentum factor. Interestingly, the value and momentum factor exposures appear to be diversifying in that they present a tendency to move in opposite directions from each other over time (Asness 1997). Note also that in the most recent years of our sample period, exposure of the risk spread portfolio to the value factor approached all-time lows, whereas the opposite was true for exposure to the momentum factor, which neared all-time highs. That is, as of the end of 2012, the relationship of the low-risk portfolio to the value factor was very strongly negative but its relationship to the momentum factor was strongly positive. With respect to the value factor, this change represents a complete reversal of the low-risk portfolio's strong positive relationship since the 1980s up until the recent financial crisis. Regarding MOM, the correlation with the zero-cost risk spread portfolio tends to decrease, often becoming negative during recessions, as can be seen from the negative correlations around 2001 and in 2008. The correlation of the low-risk portfolio with momentum tends to move higher during expansions, as has been the case since 2008. This

Figure 4. Rolling Four-Year Correlations between the Low-Risk minus High-Risk Spread Portfolios and HML and MOM for Large-Cap Stocks, 1930–2012



finding makes intuitive sense given the infrequent but severe losses suffered by momentum stocks (e.g., Daniel and Moskowitz 2013). Taken together, these findings highlight the link between valuation levels, the performance of low-risk portfolios, and momentum. Only time will tell whether these recent trends in the observed correlations between the low-risk minus high-risk spread portfolio and the MOM and HML factors will revert to more typical levels or will continue to march in opposite directions.

The Longer Evidence

Given the observed relationship between low-risk minus high-risk spread portfolio performance and recessions, we deemed it important to attempt to explore these relationships during earlier recessions, especially the Great Depression. With that motivation, we extended our analysis to the beginning of the previous century, for a total of 85 years. Note that the pre-1968 data, though perhaps somewhat unreliable owing to possible survivorship bias, should still be useful for comparison purposes. As shown in **Figure 5**, consistent with our earlier findings, the B/P for high-risk large-cap stocks increases dramatically during turbulent economic times; high-risk stocks become relatively much cheaper. For **Table 5**, we followed the procedure that we used for Table 2 but now report results over a much longer period, 1930–2012 (the data period begins in 1925, with the first five years of data used for estimation). Consistent with our earlier findings, the lowest-risk stock portfolio outperforms the highest-risk stock portfolio over various investment horizons only when the period begins in the quintile with the highest B/P spread (low risk initiates the period at a B/P discount).

Figure 5. Evolution of Average B/P for Select Beta-Focused Portfolios and the Market, 1930–2012

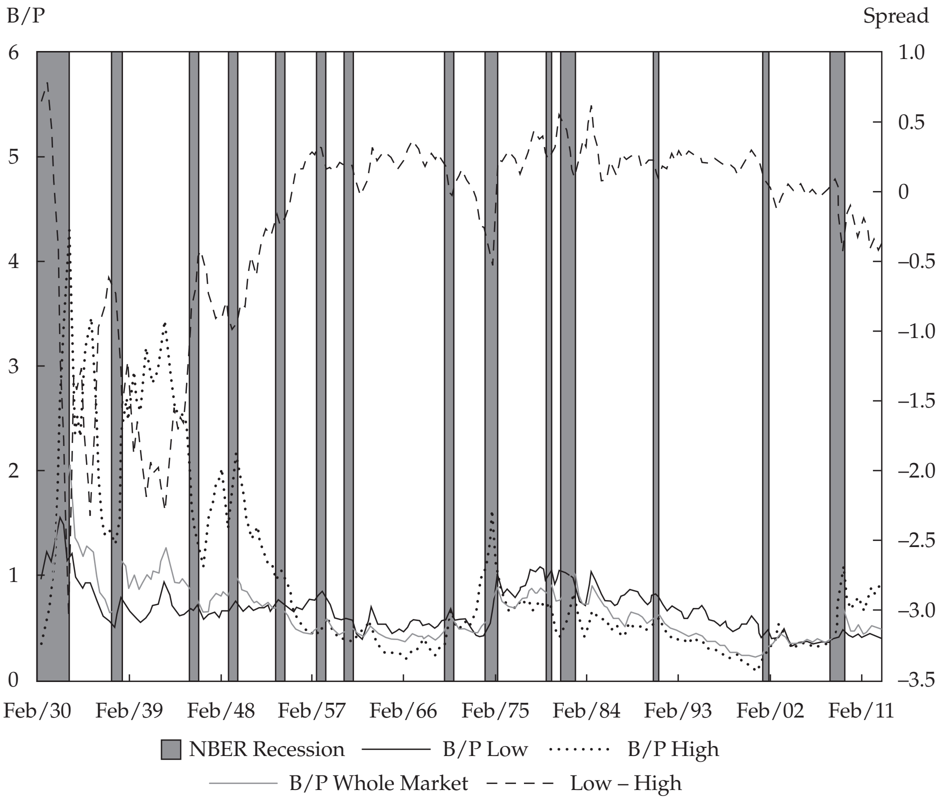


Table 5. Returns on Beta Spread Portfolios by B/P Spread, 1930–2012

B/P Spread Quintile			Future Cumulative Return on Low-Beta – High-Beta Strategy by B/P Spread Quintile (periods beyond *t* + 12 annualized)					
Quintile	From	To	*t* + 1	*t* + 12	*t* + 24	*t* + 36	*t* + 48	*t* + 60
1 (low)	–2.79	–0.65	–0.97%	–10.68%	–10.17%	–10.20%	–9.55%	–8.89%
2	–0.65	0.00	–0.38	–4.57	–5.67	–5.80	–6.15	–6.34
3	0.00	0.17	–0.24	0.22	–1.50	0.02	0.42	0.29
4	0.17	0.26	0.18	0.51	0.28	–1.02	–1.58	–0.79
5 (high)	0.26	0.72	0.62	7.44	2.84	1.57	1.83	1.28

Conclusion

Prior research on the low-volatility anomaly has focused on cross-sectional evidence emphasizing that low-volatility stocks outperform high-volatility stocks. We have extended this research by examining the time-varying performance of, and the influence of well-known investment factors on, the low-risk strategy and have included a beta-neutral low-risk strategy of practical relevance. We have done so primarily by reporting a strong dynamic link between the performance of low-volatility strategies and initial valuations—investigating the performance of both a zero-cost and a beta-neutral low-volatility strategy—and by reporting an important connection between the strategy and both economic activity and momentum.

In putting together our study's findings, an interesting picture has emerged. The performance of low-risk strategies is time varying and depends on initial valuation (like any other strategy), and low-risk strategy performance appears to be related to the well-known style and momentum factors. In other words, low-risk stocks tend to outperform high-risk stocks but are most likely to do so when initial valuation levels favor low-risk stocks. Thus, investment success seems to depend importantly on the price paid. In addition, once the well-known cross-sectional factors of style and momentum are controlled for, alphas and information ratios for our two long–short portfolio strategies decline.

We have shown that there have been extended periods over the last 85 years when high-risk stocks have cumulatively outperformed low-risk stocks. These periods have also tended to coincide, to some degree, with economic cycles. Put differently, low-risk strategies have historically outperformed more reliably if implemented when low-beta stocks have exhibited relatively high B/P levels and even more so if they have subsequently loaded positively on momentum. Interestingly, our beta-neutral portfolio experienced no negative returns for any holding period, no matter the starting valuation.

Taken together, our findings indicate that low-risk investing does indeed have worth, but investors should consider how valuation and momentum interact with low-risk portfolio performance over time. Although one should be wary of making predictions on the basis of past events, our findings suggest that the performance (like that of any quantitative investment strategy) of low-beta stocks relative to high-risk stocks relates importantly to macroeconomic, market, and valuation factors. The historical differences in performance of our two low-risk portfolio strategies also suggest the importance of careful portfolio construction. The practical implication of our results is that the performance of low-risk strategies is influenced by the approach to constructing the low-risk portfolio strategy and by time-varying exposure to the market environment and valuation premiums. Investors can benefit from a fuller understanding of how these factors may influence future performance of low-risk portfolio strategies.

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Notes

1. See Andrew Blackman, "High Hopes for 'Low Volatility' Funds," *Wall Street Journal* (6 April 2014): http://online.wsj.com/news/article_email/SB10001424052702303824204579421602356517282-lMyQjAxMTA0MDAwNzEwNDcyWj.
2. We obtained similar results when we used idiosyncratic volatility as our risk measure but do not report the results here owing to space constraints.
3. See http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.
4. So-called volatility drag creates a headwind that probably contributes to the underperformance of higher-risk quintiles over time. As is well known, the geometric return compounds at a rate roughly equivalent to the arithmetic return minus half the variance of returns.
5. We do not claim that the data follow a Gaussian distribution, but we nonetheless follow the extant literature in interpreting *t*-statistics in this manner. The well-known caveats to such an interpretation apply.
6. Table 1 reports raw returns, ignoring any return from short cash proceeds associated with our long–short portfolios. However, we appropriately used excess returns in reporting all Sharpe ratios.
7. For more on the efficacy of extracting the low-volatility anomalous returns, see Li et al. (2014), who provided a detailed look in order to include various periods, market-capitalization groupings, and weighting schemes.
8. Our approach is much simpler than that of Frazzini and Pedersen (2014), whose betting-against-beta (BAB) strategy is dynamically managed to keep the portfolio at a beta of roughly 0. A BAB portfolio holds low-beta assets, leveraged to a beta of 1, and shorts high-beta assets, deleveraged to a beta of 1.
9. We report excess returns for both of our long–short portfolios. For the regular long–short portfolio, we start with a portfolio that is 100% long and 100% short. The return on this portfolio equals 100% × Long portfolio return – 100% × Short portfolio return + 100% × Cash return on short proceeds. This approach makes the excess portfolio return, or the portfolio return in excess of the return on three-month T-bills, equal to 100% × (Long portfolio return – Short portfolio return). Given that our beta-neutral portfolio is 100% long and 25% short, the return on this portfolio equals 100% × Long portfolio return – 25% × Short portfolio return + 25% × Cash return on short proceeds, with excess returns on the beta-neutral portfolio equal to 100%

× Long portfolio return – 25% × Short portfolio return – 75% × Cash return.

10. We rebalance monthly, consistent with the extant literature that posits a low-risk alpha associated with a monthly rebalancing process. We use the term "investment horizon" and not "holding period" because we are rebalancing our risk quintile portfolios each month on the basis of the level of beta for each stock instead of simply buying and holding that portfolio. We believe this process better reflects the practice of portfolio construction.

11. Clarke et al. (2010) used the Fama and French (1993) methodology to create a volatility factor. They reported that the volatility factor is an important risk factor, although it is correlated with MKT (market risk premium) and SMB. On the basis of their analysis, the relationship between the volatility factor and HML is unstable and has turned negative in recent decades. Our findings extend and clarify theirs, particularly regarding the interactions between low-risk strategies and HML and UMD (up minus down).

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