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KEY FINDINGS

- The risk of concentrated holdings is dominated by the stock-specific component, which cannot be hedged with unrelated stocks.
- As a result, while the completion portfolio approach to hedging the risk of concentrated stock appears conceptually sound, it offers limited practical risk-reduction benefits.
- Selling the concentrated position quickly and tax efficiently is the most effective risk-mitigation strategy. Tax-aware strategies, particularly tax-aware long-short strategies, can be highly valuable in this respect.

ABSTRACT

This article investigates the most effective ways to manage the risk of concentrated stock positions. While the completion portfolio approach to hedging the risk of concentrated stock appears conceptually sound, the empirical evidence shows that it offers limited practical risk-reduction benefits. The risk of concentrated holdings is dominated by the stock-specific component, which cannot be hedged with unrelated stocks. We find that simple sector-exclusion completion portfolios are generally ineffective as concentrated stock hedges. Optimized completion portfolios can hedge common-factor risk to a degree, especially when shorting is used. However, their overall impact on forecast risk is small, and on realized risk, negligible. Selling the concentrated position quickly and tax efficiently is the most effective risk-mitigation strategy. Tax-aware strategies, particularly tax-aware long-short strategies, can be highly valuable in this respect. The article concludes that investors and their advisors should temper their expectations about the risk-hedging ability of completion portfolios and prioritize the tax-aware aspect of the strategies to facilitate the tax-efficient liquidation of concentrated holdings.

When it comes to a single stock's performance, the story is always different, but regrettably, the outcome is often the same. Most stocks underperform the market-capitalization-weighted portfolio (Bessembinder 2018). Five-year winners tend to significantly underperform the median stock over the following 10 years (Petajisto 2023). Recommendations from Wall Street research desks are not much help either, as they usually miss the mark (Cembalest 2024).[1]

[1] Cembalest (2024) shows that, of the stocks that experienced catastrophic losses—defined as a 70% drop from peak valuation without subsequent recovery—80% had a "Strong Buy" Street consensus recommendation, and, of the remaining 20%, almost all had a "Buy" recommendation at the peak.

In hindsight, there is always a "plausible" explanation for why a given stock crashed, some "obvious" signs that were overlooked. In some cases, it's the macroeconomic environment—COVID, trade wars, or monetary policy that is either too tight or too loose. In other cases, it's an industry-wide shock. Still, in others, it's company management neglecting proper risk controls or failing to foresee demand shocks for the product.

Whether these post-mortems are true or not, single-stock underperformance boils down to fundamental statistics: most single stocks have high volatility, and the volatility drag, over time, shifts the center of the distribution of stock price toward zero; the longer the holding period, the greater the shift toward zero (Sosner 2022). This theoretical result is supported by data (Petajisto 2023).

In light of this evidence, reducing concentrated stock risk should be the highest priority for concentrated stock investors. Moreover, since the likelihood of losses increases with the passage of time, the speed of risk reduction matters. In this article, we examine one technique for reducing concentrated stock risk: a completion portfolio approach.

The completion portfolio approach to diversifying concentrated, low-basis stock positions is commonly described as follows:[2] First, the investor injects capital into the diversification process—by selling part of the concentrated position, by using preexisting capital, or both. This capital is then invested in a portfolio with two objectives: to reduce the risk of the concentrated position, and to harvest losses to facilitate further selling of the position. Some risk reduction is expected to be achieved by hedging the risk exposures of the concentrated position—for example, by underweighting the sector to which the concentrated stock belongs. As losses are realized, the stock is gradually sold, and the sale proceeds continue to be invested in the completion portfolio, with the ultimate objective of replicating a benchmark index portfolio.

The benefit of systematically realizing capital losses to offset the gains from selling appreciated assets has been well documented and well understood for at least a quarter of a century (e.g., Stein and Narasimhan 1999 is among the earliest sources). Prior literature provides conclusive evidence that strategies systematically realizing losses can create valuable tax benefits.[3] The focus of this article is to determine whether completion portfolios can offer the purported risk-hedging benefits to concentrated stock investors.

Conceptually, "wrapping" the diversified part of the portfolio around the concentrated stock makes sense. Constructing a completion portfolio that can hedge exposures to the stock's sector and other risk factors seems like a prudent portfolio management decision. That said, the question is whether the data show evidence of diversified portfolios meaningfully hedging concentrated stock risk.

To preview our results, we find little empirical evidence that the completion portfolio approach helps hedge the risk of concentrated stock positions. While conceptually prudent, it is ineffective in practice. In contrast, selling a concentrated position quickly and tax efficiently is highly beneficial for de-risking the overall investment portfolio. As a result, the ability of the diversified part of the portfolio to realize capital losses is crucial. Obviously, realizing losses is of little value if the diversified portfolio loses money while doing so. This is precisely where tax-aware long-short factor strategies prove to be powerful; they have been shown to be highly effective at balancing pre-tax alpha generation with capital loss realization.

[2] Of course, the low-basis stock can simply be sold outright, but many investors may find the tax burden from an outright sale to be unacceptably high.

[3] See Sosner, Gromis, and Krasner (2022), Liberman et al. (2023), and numerous citations therein.

THE COMPLETION PORTFOLIO APPROACH TO DIVERSIFYING CONCENTRATED STOCK

Description of the Approach

The completion portfolio approach to diversifying concentrated stock dates back at least to Stein et al. (2000) and has been reiterated by many others, including Welch (2002) and, more recently, Bouchey (2024). It is currently being highlighted by numerous financial service providers, such as Morgan Stanley (2023), Vanguard (2023), Goldman Sachs (2024), and Parametric (Milleson 2004), among others.

The concentrated stock can be part of a larger pool of assets owned by the investor, which may include other stocks and cash available for investment (see, e.g., Morgan Stanley 2023; Vanguard 2023; and Milleson 2004).

The approach is implemented as follows: The concentrated stock is sold gradually via staged sales, possibly over multiple years. A completion portfolio is funded by combining the proceeds from the initial sale of the stock with any existing diversified portfolio positions and previously available cash. The completion portfolio is constructed to hedge the outsized risk exposures of the remaining concentrated stock position. Proceeds from the staged sales are reinvested into the completion portfolio. The staged sales provide time for the completion portfolio to realize capital losses, which can offset capital gains from selling the stock.

Direct indexing is typically a go-to strategy for implementing this approach. To hedge the exposures of the concentrated stock position, the direct-indexing completion portfolio may, for example, exclude the sector to which the stock belongs (see Morgan Stanley 2023; Vanguard 2023; Bouchey 2024; Goldman Sachs 2024; and Milleson 2004). If an optimization-based approach is used, the completion portfolio may utilize a risk model to hedge the stock's factor exposures more broadly than just its sector exposure (see Bouchey 2024). For example, if the stock loads positively on the momentum risk factor, the completion portfolio may have a negative loading on the momentum factor. The direct-indexing completion portfolio then begins harvesting losses to reduce the tax burden from the staged sale of the concentrated stock.[4]

Potential Shortcomings of the Approach

While this approach is generally prudent and logical, there are several potential concerns regarding its practical efficacy. Here, we provide a summary of our considerations and defer a detailed analysis to the Appendix.

The prevalence of stock-specific risk. A stock's risk comprises two main components: common-factor risk and stock-specific risk. A stock's common-factor risk, such as industry, sector, and style-factor risk, can be hedged, at least in theory, using other unrelated stocks. In contrast, stock-specific risk cannot be hedged with unrelated stocks and only can be diversified by reducing the stock's weight in the overall portfolio. To add insult to injury, as a stock's volatility increases—and therefore the need for de-risking becomes more urgent—the stock-specific risk becomes a larger contributor to the stock's total risk (see the Appendix). As a result, we expect the risk hedging provided by completion portfolios to be largely ineffective.

Sector exclusion. Although stocks tend to have higher correlations within a sector rather than across sectors, the difference between within-sector and between-sector pairwise stock return correlations is not substantial; the former is 0.30 and the latter

[4] As clarified in Krasner and Sosner (2024), loss harvesting is a trading activity that is primarily motivated by tax considerations.

is 0.24.[5] This means that the sector component, although present, is not a very substantial driver of single-stock returns. Indeed, stocks within the same sector, and even stocks with very similar products and services, can exhibit vastly different performance due to specific management decisions (see examples in Cembalest 2024). In sum, we expect that sector exclusions should be ineffective at hedging the risk of concentrated stock.

Optimized completion portfolio. One could argue that excluding a sector might be a tool that is too blunt for the job. What about using a risk model to build a completion portfolio that hedges not only the sector but also other risk-factor exposures of the stock? Such a portfolio might underweight the stock's sector and industry, and have style-factor exposures (e.g., value, momentum, quality, and size) with signs opposite to those of the stock. Ex-ante, such a portfolio is the best portfolio for reducing the stock's systematic risk associated with its sector, industry, and style-factor exposures. However, there are two potential problems with this approach. First, industry and style-factor risk contributions are relatively small components of the total single-stock risk, which is dominated by stock-specific risk (see the Appendix). Second, the ex-ante risk reduction predicted by the risk model may not materialize ex-post in the realized risk of the portfolio.

Loss realization by the completion portfolio. Losses realized by the completion portfolio could be highly valuable for offsetting the gains realized on selling the concentrated position. However, losses realized by direct-indexing strategies are typically small (see, e.g., Goldberg, Cai, and Hand 2022; Goldberg, Cai, and Selwitz 2022; Israelov and Lu 2023; and Liberman et al. 2023). Furthermore, losses realized by direct-indexing strategies are highly variable depending on the market environment (see, e.g., Sosner, Gromis, and Krasner 2022; Israelov and Lu 2023; and Liberman et al. 2023). In other words, losses realized by direct-indexing strategies are both low on average and highly uncertain. An investor looking for tax-efficient diversification of concentrated risk with direct indexing should be aware of these limitations and prepared to either pay a significant amount in taxes (see, e.g., Goldberg, Cai, and Selwitz 2022) or commit a large sum of additional capital to the direct-indexing strategy. Compared to long-only direct indexing, tax-aware long-short strategies can alleviate the significant tax burden resulting from selling low-basis stock and also reduce the need for additional capital. The need for additional capital can be reduced because concentrated positions can be used to finance long and short extensions, as discussed in Goldberg, Cai, and Selwitz (2022) and Bouchey (2024).[6]

In the remainder of the article, we illustrate the shortcomings of the completion portfolio approach to diversifying concentrated stock. Note that our criticism is not directed at the completion portfolio approach in general.[7] Nor is it directed at long-only tax management.[8] Both can be effective tools in the appropriate situations. However, when the risk is dominated by the stock-specific component and the urgency to reduce

[5] In analysis not reported here for the sake of brevity, using the Barra USE3L risk model, we computed average within-sector and between-sector pairwise stock return correlations for the sample of stocks described in the methodology section.

[6] Liberman and Sosner (2025b) and Liberman, Sosner, and Maron (2025) demonstrate the efficacy of tax-aware long-short factor strategies for diversifying concentrated stock when they are combined with variable prepaid forwards (VPFs) or charitable remainder unitrusts (CRUTs).

[7] In many cases, completing exposures missing from the investor's portfolio can be quite valuable. For example, Bouchey and Fong (2023) show an example of a completion portfolio providing a missing exposure to Japanese equities in a portfolio tilted toward European equities due to a manager with greater alpha-generating opportunities in Europe.

[8] Long-only tax-aware portfolio management can be valuable in the contexts of global tactical asset allocation (Aked et al. 2019) and core-and-satellite portfolio structure (Quisenberry 2003 and Bouchey and Pritamani 2017).

risk is high, using long-only tax-managed completion portfolios is likely not the most effective tool for the job.

METHODOLOGY

Sampling Single Stocks

Every December from 2000 to 2021—22 Decembers in total—we sample 500 single stocks from the Russell 3000 Index universe. We first partition the index constituents into quintiles of approximately 600 stocks each, by stock-level forecast volatility, where the forecast volatility is measured using the Barra USE3L risk model. We then randomly draw 100 stocks from each volatility quintile, resulting in 500 stocks in each December sample. Five hundred stocks over 22 Decembers gives us 11,000 observations.

Portfolio Construction

For each of the 11,000 observations, we construct portfolios combining concentrated stock with a diversified portfolio. We vary the concentrated stock weight in the portfolio from 20% to 80% in 20% increments. We model three types of portfolios, a summary of which is provided in Exhibit 1.

The first portfolio type is a simple combination of a concentrated stock and the Russell 3000 Index. For example, the stock makes up 20% of the total portfolio, while the Russell 3000 Index portfolio comprises the remaining 80%. This creates a stock-oblivious benchmark for completion portfolios.

The next two types of portfolios are completion portfolios that take concentrated stock into account: sector-exclusion portfolios and optimized completion portfolios. For the optimized completion portfolios, we construct both long-only and long-short portfolios. Let's discuss them one at a time.

Sector-exclusion portfolios only consider the sector to which the stock belongs. Stocks belonging to that sector are dropped from the index portfolio, and the remaining portfolio weights are rescaled to sum to 100%. The sector-exclusion portfolio is then combined with the concentrated stock using the allocation weights described earlier.

EXHIBIT 1

Portfolio Types and Their Characteristics

	Portfolio Type						
			Optimized Completion Portfolio				
	Benchmark Index	**Sector Exclusion**	**Long-Only**	**125/25**	**150/50**	**200/100**	**250/150**
Allocation Weights	Concentrated stock: 20% to 80% in 20% increments; diversified portfolio: remaining weight						
Diversified Portfolio	Passive index	Passive index	Optimized portfolio				
Exclusions	None	Sector	None				
Optimization Settings							
Objective			Minimize Tracking Error				
Sum of Weights			100%				
Sum of Long Weights			100%	125%	150%	200%	250%
Sum of Short Weights			0%	25%	50%	100%	150%

Optimized portfolios are constructed with the main objective of reducing tracking error (TE) without selling any of the concentrated stock. In the Appendix, we provide further details on the levels of TE before the diversified portfolio is optimized for TE reduction. We construct long-only and long-short optimized completion portfolios. The optimization routine is very simple: minimize TE subject to the constraint that the sum of the portfolio's weights equals to one. For the long-only portfolio, all the weights are constrained to be positive. For the long-short portfolios, the sums of long and short positions are constrained to be a certain percentage of the portfolio NAV. We consider four long-short portfolio leverage levels: 125/25, 150/50, 200/100, and 250/150.

The concentrated stock is excluded from the diversified portfolio. There are no other optimization penalties or constraints, such as transaction or financing cost penalties, trade or position size constraints, or industry or sector exposure constraints.

Note that we only model completion portfolios at the inception of the de-risking process. This allows the completion portfolios to focus solely on pursuing the concentrated stock hedging objective. At later dates, tax-efficient rebalancing is likely to impose onerous constraints on the completion portfolio composition, detracting from that objective.

Realized Tracking Error Calculation

To compute realized tracking error, we track the daily active returns—portfolio returns in excess of the benchmark returns—for each constructed portfolio for 60 business days after formation, without changing the portfolio composition. We then compute the annualized standard deviation of the resulting 60-day daily active return series.

FORECAST RISK REDUCTION OFFERED BY COMPLETION PORTFOLIOS

Sector Exclusion

We start by measuring the forecast tracking error (TE) of portfolios with different allocations to a concentrated stock and a diversified portfolio. We vary the allocation to the stock from 20% to 80% in 20% intervals. The analysis does not depend on whether the additional capital for funding the diversified portfolio comes from new capital or from the proceeds of selling part of the stock position. We consider two diversified portfolio types: a benchmark index portfolio oblivious to the concentrated stock and a completion portfolio that excludes the sector to which the stock belongs.

Exhibit 2, Panel A, shows the distributions of forecast TE for alternative allocations to the concentrated stock and the diversified portfolio.[9] While having less stock in the portfolio significantly reduces the TE, the differences in TE between the stock-oblivious benchmark index portfolio and the portfolio excluding the stock's sector are hardly noticeable.

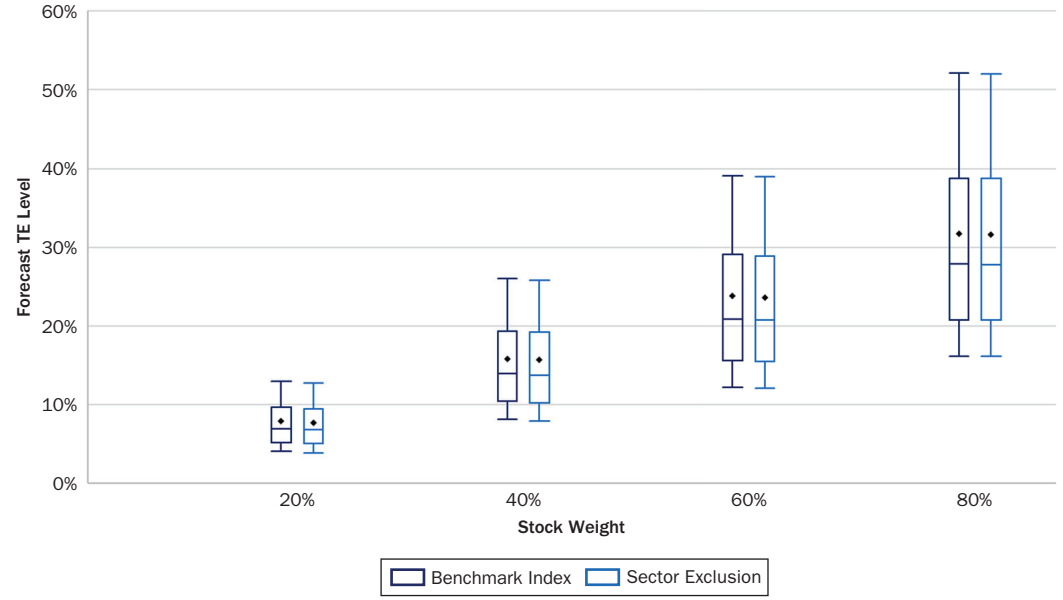
Exhibit 2, Panel B, zooms in on the forecast TE differences between the two types of diversified portfolios. Compared to the stock-oblivious index portfolio, excluding the sector results in a reduction ranging from 4 to 11 basis points for the median TE and from 7 to 19 basis points for the mean TE. These TE reduction values are negligibly small compared to the levels of forecast TE, which are measured in multiple percentage points.

[9]The distribution here, and in all the following charts, is computed over 11,000 observations, as explained in the methodology section.

EXHIBIT 2

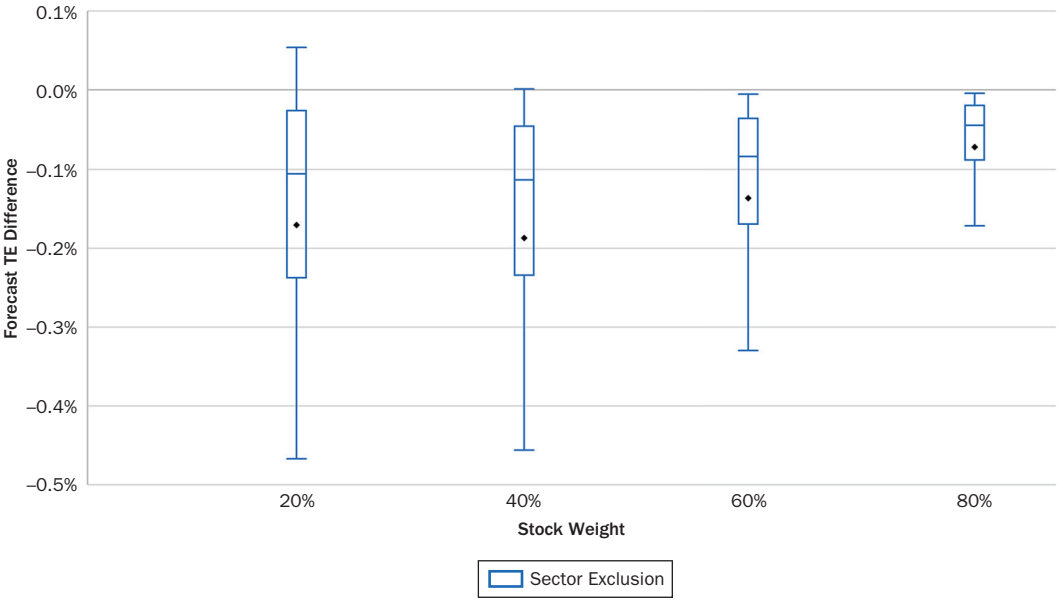
Forecast TE of Sector-Exclusion Portfolios versus Benchmark Index Portfolio

Panel A: Side-by-Side Forecast TE Comparison



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	7.9%	7.0%	15.9%	13.9%	23.8%	20.9%	31.7%	27.8%
Sector Exclusion	7.8%	6.8%	15.7%	13.8%	23.6%	20.8%	31.6%	27.8%

Panel B: Forecast TE Difference: Sector Exclusion Minus Benchmark Index



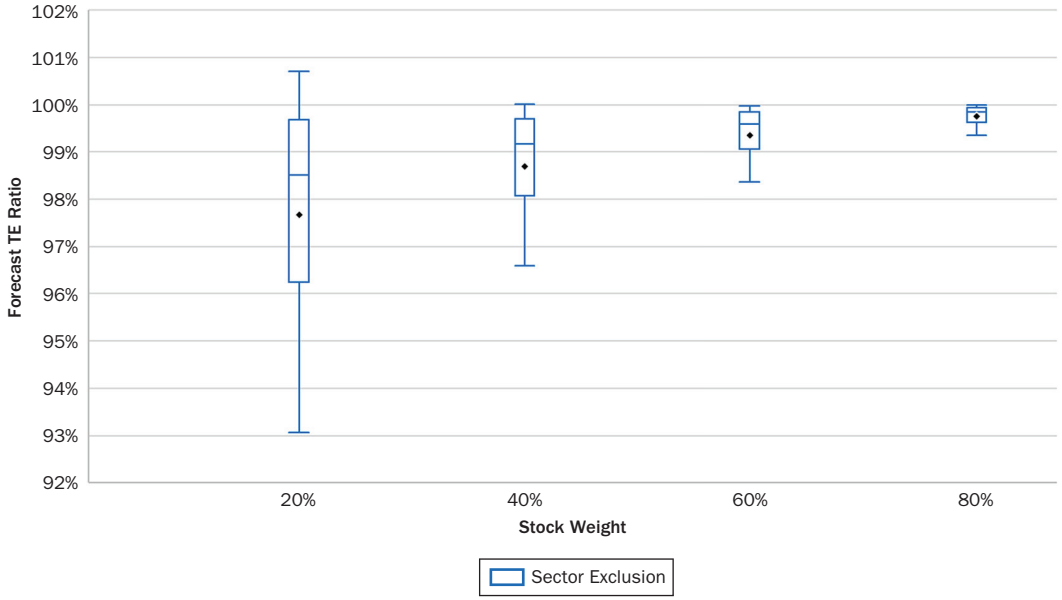
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	–0.17%	–0.11%	–0.19%	–0.11%	–0.14%	–0.08%	–0.07%	–0.04%

(continued)

EXHIBIT 2 *(continued)*

Forecast TE of Sector-Exclusion Portfolios versus Benchmark Index Portfolio

Panel C: Forecast TE Ratio: Sector Exclusion Divided by Benchmark Index



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	97.7%	98.5%	98.7%	99.2%	99.4%	99.6%	99.7%	99.8%

NOTES: In this exhibit, the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

Exhibit 2, Panel C, shows how small the impact of sector exclusion is by measuring the forecast TE ratio for the two types of diversified portfolios. In the vast majority of cases, the TE in the two diversified portfolio scenarios is practically identical.

In sum, excluding the stock's sector is directionally correct, but its impact on forecast TE is negligible, which makes the sector exclusion practically ineffective.

Optimized Completion Portfolio

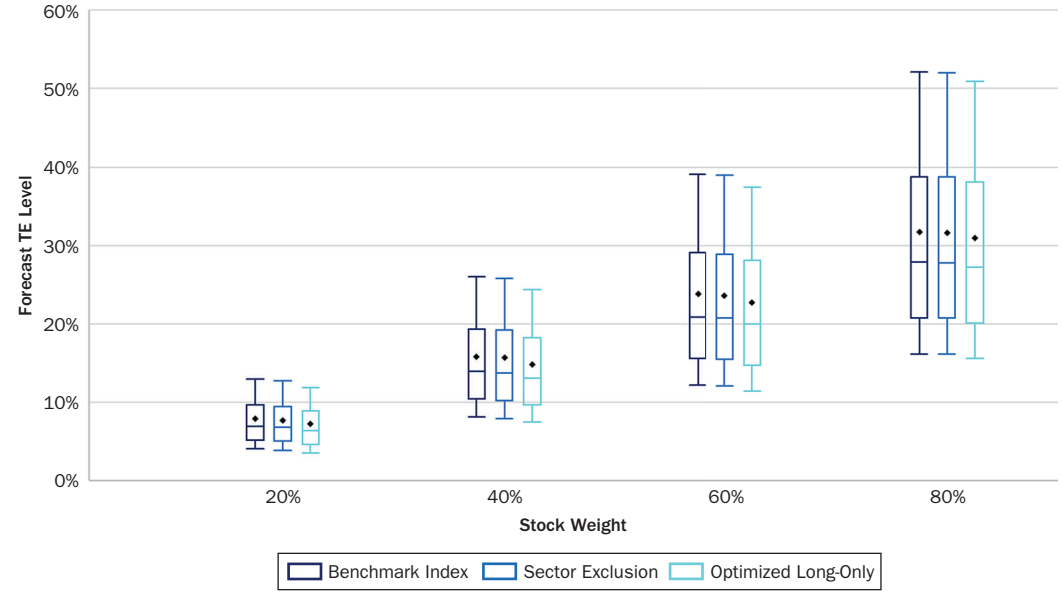
We now add the third portfolio type—the optimized completion portfolio—to our analysis of forecast TE reduction. Exhibit 3, Panel A, shows the distribution of forecast TE for all three diversified portfolio types—stock-oblivious index, sector-exclusion, and optimized portfolio. The difference in TE across the scenarios is small. Nonetheless, it is possible to discern that the TE reduction is marginally stronger for the optimized portfolio approach than for sector exclusion.

Exhibit 3, Panel B, shows the distribution of forecast TE differences from the stock-oblivious index for the sector-exclusion and optimized approaches. The median TE reduction for the optimized completion portfolio is between 53 and 76 basis points, and the mean TE reduction reaches 1% for the concentrated stock weights of 40% and 60%. Note that, similar to sector exclusion, the TE reduction for optimized portfolios is negligible in comparison to the levels of forecast TE seen in Exhibit 3, Panel A.

EXHIBIT 3

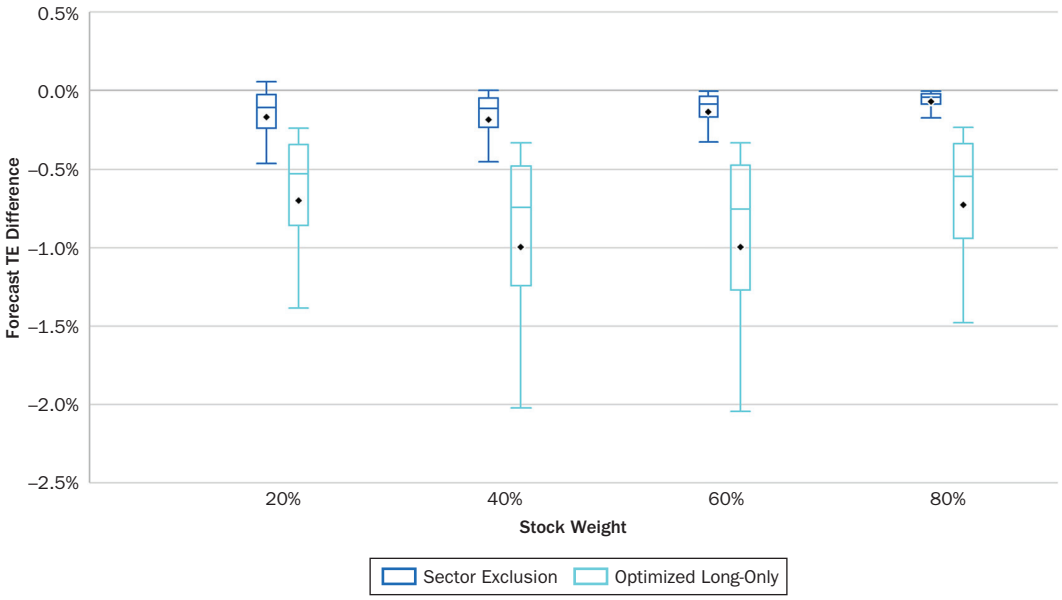
Forecast TEs of Sector-Exclusion and Optimized Portfolios versus Benchmark Index Portfolio

Panel A: Side-by-Side Forecast TE Comparison



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	7.9%	7.0%	15.9%	13.9%	23.8%	20.9%	31.7%	27.8%
Sector Exclusion	7.8%	6.8%	15.7%	13.8%	23.6%	20.8%	31.6%	27.8%
Optimized Long-Only	7.2%	6.4%	14.9%	13.1%	22.8%	20.0%	31.0%	27.2%

Panel B: Forecast TE Difference: Sector Exclusion and Optimized Minus Benchmark Index



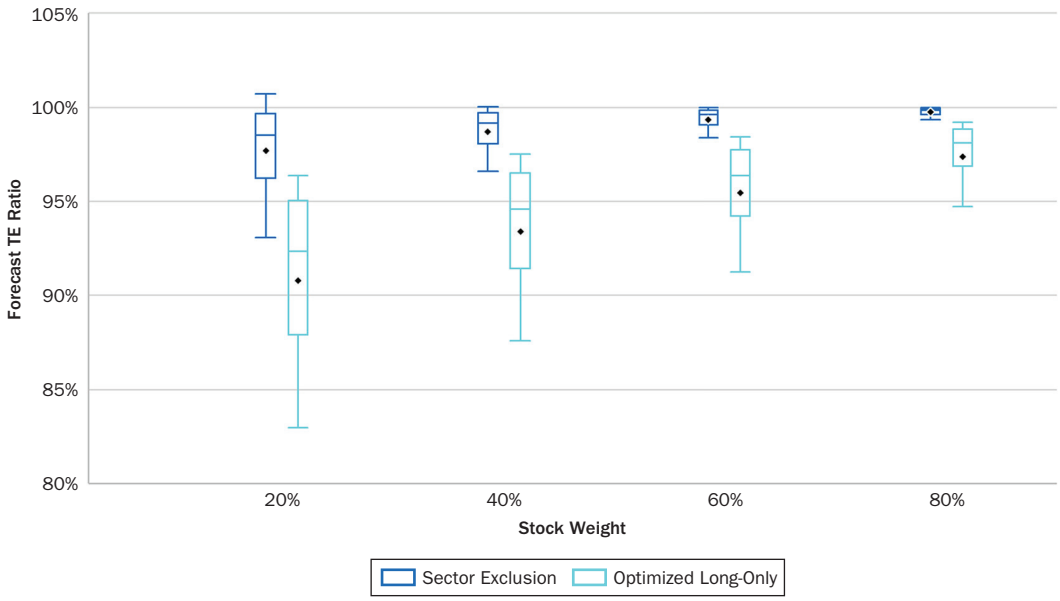
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	–0.17%	–0.11%	–0.19%	–0.11%	–0.14%	–0.08%	–0.07%	–0.04%
Optimized Long-Only	–0.70%	–0.53%	–1.00%	–0.74%	–0.99%	–0.76%	–0.73%	–0.55%

(continued)

EXHIBIT 3 *(continued)*

Forecast TEs of Sector-Exclusion and Optimized Portfolios versus Benchmark Index Portfolio

Panel C: Forecast TE Ratio: Sector Exclusion and Optimized Divided by Benchmark Index



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	97.7%	98.5%	98.7%	99.2%	99.4%	99.6%	99.7%	99.8%
Optimized Long-Only	90.8%	92.3%	93.4%	94.6%	95.5%	96.4%	97.4%	98.1%

NOTES: In this exhibit, the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

Exhibit 3, Panel C, shows that the ratio of forecast TE under the optimized approach to that under the stock-oblivious index is significantly lower than that of the sector exclusion to the index. Nonetheless, the mean and median risk reduction offered by the optimized approach is still quite low. Even when the stock position is only 20% of the overall portfolio, the mean and median forecast TE ratios are 90.8% and 92.3%, respectively. Note that the ratio is the lowest, meaning the relative TE reduction is the greatest, exactly when it benefits the investor the least—when the stock position is already a small fraction of the overall portfolio. This further underscores the limited effectiveness of the completion portfolio approach to hedging concentrated stock.

Drivers of Forecast Risk Reduction

By definition, the risk reduction offered by completion portfolios should be primarily driven by a reduction in the common-factor risk. We observe this in Exhibit 4.

Exhibit 4, Panel A, shows the reduction in the common-factor risk. Not surprisingly, the optimized approach, which accounts for all risk factors in the risk model and not just sector risk, achieves a greater reduction in common-factor risk than sector exclusion.

Exhibit 4, Panel B, confirms that the stock-specific risk contribution to the forecast TE is not affected by either the sector-exclusion or the optimized approach.

It is also instructive to compare the relative sizes of risk contributions in Panels A and B. Not only is the stock-specific contribution to the forecast TE unaffected by completion portfolios, but it is also substantially higher than the common-factor contribution.

Recap of Forecast Risk Reduction Experiments

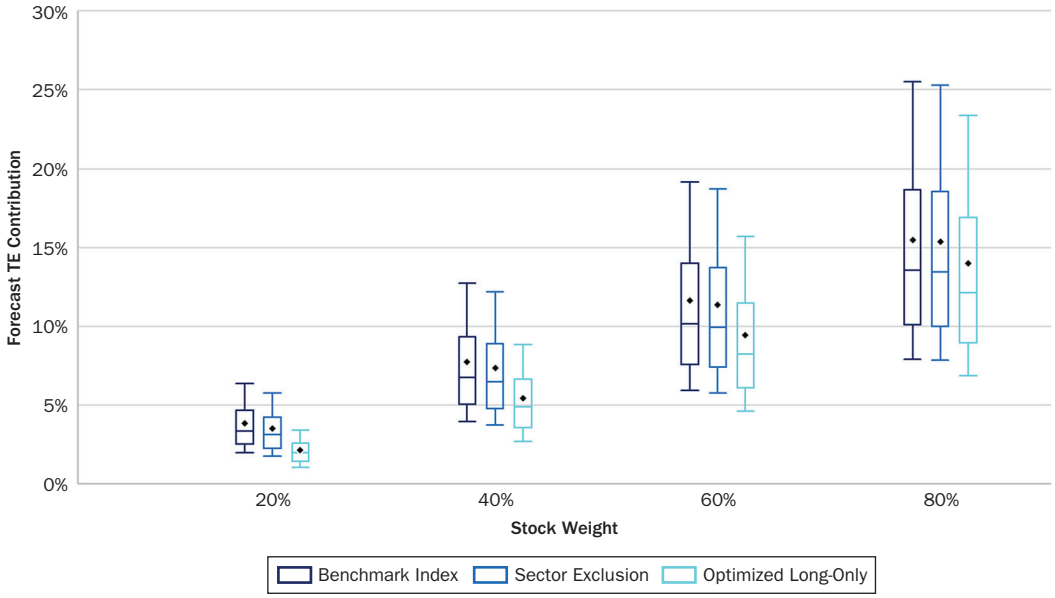
Stock-specific risk is the largest driver of a single stock's volatility. This component of risk is impossible to hedge with other unrelated stocks in a completion portfolio. It can only be reduced by making the concentrated position a smaller fraction of the overall stock portfolio.[10]

Completion portfolios that exclude the sector to which the concentrated stock belongs are only marginally more effective at reducing the forecast risk than a stock-oblivious benchmark index portfolio. Optimized completion portfolios fare a little better than sector-exclusion portfolios. However, their effectiveness at concentrated risk reduction is still very limited.

EXHIBIT 4

Forecast TE Decomposition to Common-Factor and Stock-Specific Components

Panel A: Common-Factor Forecast TE Contribution



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	3.9%	3.4%	7.7%	6.8%	11.6%	10.2%	15.5%	13.5%
Sector Exclusion	3.5%	3.1%	7.4%	6.5%	11.4%	9.9%	15.4%	13.4%
Optimized Long-Only	2.1%	2.0%	5.5%	4.9%	9.5%	8.3%	14.0%	12.1%

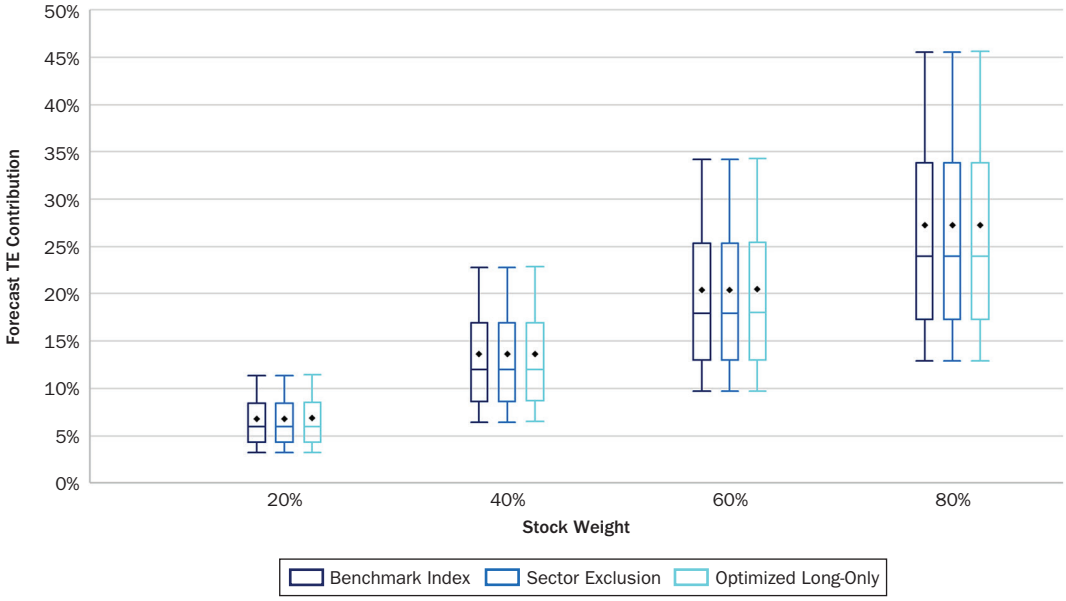
(continued)

[10] An investor can also engage in derivatives hedging transactions, such as variable prepaid forwards, in which case the investor must analyze both the economics and tax implications of such hedging (see, e.g., Liberman and Sosner 2025a).

EXHIBIT 4 *(continued)*

Forecast TE Decomposition to Common-Factor and Stock-Specific Components

Panel B: Stock-Specific Forecast TE Contribution



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	6.8%	6.0%	13.6%	12.0%	20.4%	18.0%	27.3%	24.0%
Sector Exclusion	6.8%	6.0%	13.6%	12.0%	20.4%	18.0%	27.3%	24.0%
Optimized Long-Only	6.8%	6.0%	13.7%	12.0%	20.5%	18.0%	27.3%	24.0%

NOTES: In this exhibit the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

REALIZED RISK REDUCTION OFFERED BY COMPLETION PORTFOLIOS

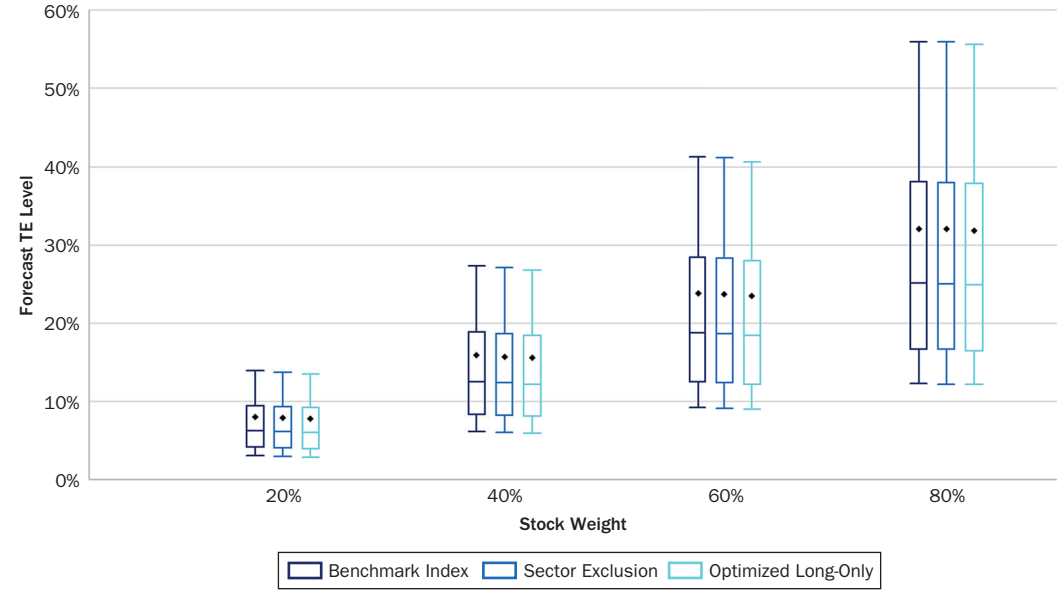
Thus far, we have only considered forecast risk reduction. However, like any forecast, risk forecasts generated by risk models are merely informed guesses about future risk levels. Realized outcomes may differ significantly from those forecasts. The optimized approach, which relies on the risk model's predictions for risk minimization, could be particularly susceptible to overfitting the projected reduction in risk. In this section, we explore whether a reduction in forecast TE offered by sector-exclusion and optimized completion portfolios translates into a reduction in realized volatility.

Exhibit 5, Panel A, shows the realized TE under our three alternative scenarios for the diversified part of the overall portfolio—stock-oblivious benchmark index, sector exclusion, and optimized completion portfolio. Any differences observed in Exhibit 3 between the stock-oblivious index and one of the two completion approaches, particularly the optimized approach, disappear when realized TE is considered. Only one pattern we saw for forecast TEs remains strongly represented in the data: less concentrated stock in the portfolio unequivocally translates into lower TE.

EXHIBIT 5

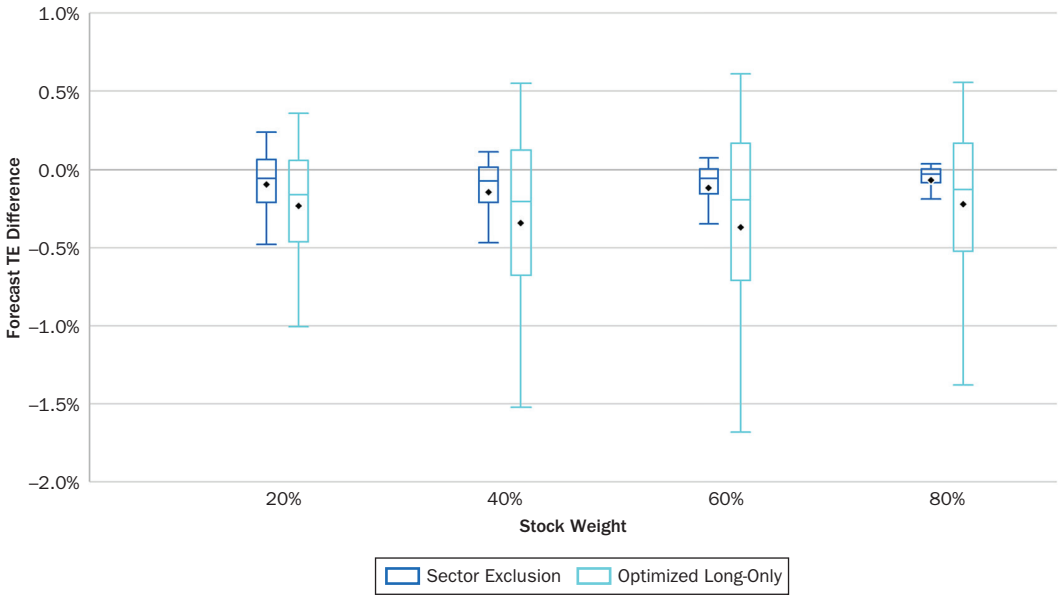
Realized TEs of Sector-Exclusion and Optimized Portfolios versus Benchmark Index Portfolio

Panel A: Side-by-Side Realized TE Comparison



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	8.1%	6.3%	15.9%	12.5%	23.9%	18.8%	32.1%	25.1%
Sector Exclusion	8.0%	6.2%	15.8%	12.4%	23.8%	18.7%	32.0%	25.1%
Optimized Long-Only	7.8%	6.0%	15.6%	12.2%	23.5%	18.4%	31.9%	24.9%

Panel B: Realized TE Difference: Sector Exclusion and Optimized Minus Benchmark Index



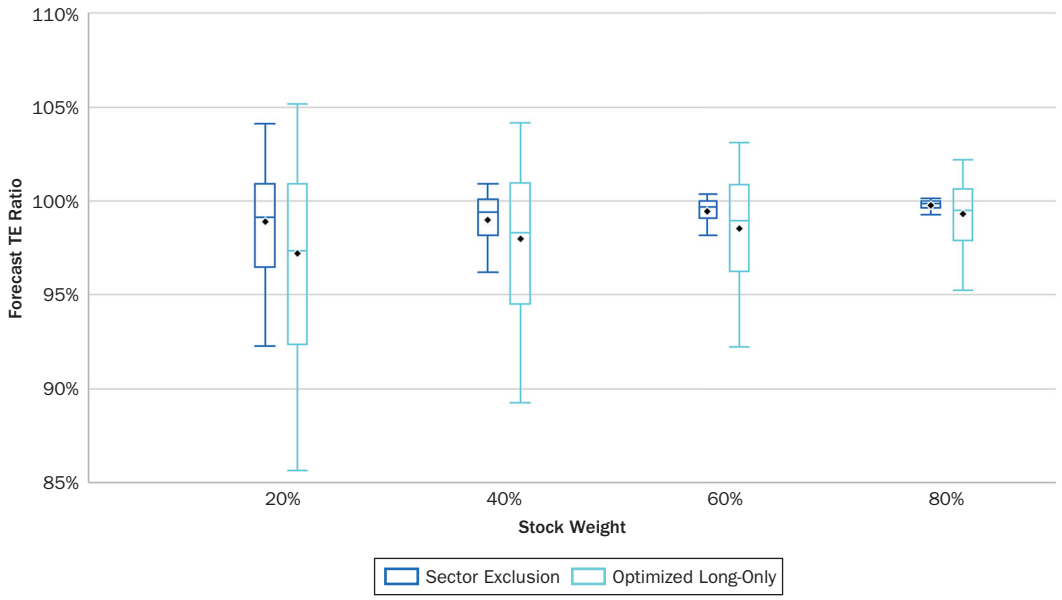
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	–0.10%	–0.06%	–0.15%	–0.07%	–0.12%	–0.06%	–0.07%	–0.03%
Optimized Long-Only	–0.23%	–0.16%	–0.34%	–0.20%	–0.37%	–0.20%	–0.22%	–0.13%

(continued)

EXHIBIT 5 *(continued)*

Realized TEs of Sector-Exclusion and Optimized Portfolios versus Benchmark Index Portfolio

Panel C: Realized TE Ratio: Sector Exclusion and Optimized Divided by Benchmark Index



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	98.9%	99.2%	99.0%	99.4%	99.5%	99.7%	99.8%	99.9%
Optimized Long-Only	97.2%	97.4%	98.0%	98.3%	98.6%	98.9%	99.3%	99.5%

NOTES: In this exhibit, the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

Exhibit 5, Panels B and C, allow us to further zoom in on the realized TE differences and ratios of completion portfolios in relation to the stock-oblivious benchmark index portfolios. Compared to the forecast TE differences in Exhibit 3, the mean and median differences between the realized TEs of the two types of completion portfolios and the stock-oblivious index shrink toward zero, while the TE ratios shrink toward one. For the optimized portfolio, zero difference and a ratio of one are now within the 25th to 75th percentile range. This means that, in addition to being economically negligible, the realized TE reduction offered by the optimized approach also is not statistically significant; any benefits of the optimized completion approach observed in forecast TEs disappear in realized TEs.

In sum, we find no evidence that, for an average stock, completion portfolios reduce the risk of concentrated stock exposure. The only unambiguous result is that having less concentrated stock in a portfolio reduces its risk. This result is both economically strong and highly statistically significant.

Note that this analysis by no means invalidates the concept of the completion portfolio approach to de-risking concentrated stock. For example, a priori, it is hard to argue that an investor whose net worth is highly concentrated in a technology stock should hold a diversified portfolio oblivious to this fact, and thus amplify the overall portfolio's exposure to the technology sector. The purpose of our analysis is to help investors and their advisors form realistic expectations about the effectiveness of the completion portfolio approach for hedging concentrated risk.

LONG-SHORT OPTIMIZED COMPLETION PORTFOLIOS

Thus far, our analysis suggests that completion portfolios generally are ineffective at hedging the risk of concentrated stock, whether they are constructed using sector exclusion or optimization. However, up to this point, we only have considered long-only portfolios. As the long-short approach gradually becomes a mainstay of tax-efficient portfolio management, we should consider it as well. As shown in Liberman et al. (2023) and Krasner and Sosner (2024), tax-aware long-short strategies can be highly effective at balancing pre-tax alpha generation with capital loss realization. Sosner and Krasner (2021) and Goldberg, Cai, and Selwitz (2022) further show that the tax-aware long-short approach can be effective at modifying equity portfolios in a tax-efficient way. Can long-short portfolios also be effective as completion portfolios for concentrated stock?

Based on our observations in previous sections, we do not have high hopes. First, most of the risk arising from concentrated positions is stock-specific. This risk can be diversified by reducing the position size in the overall portfolio, but it cannot be hedged with other unrelated stocks. Therefore, while long-short strategies may help sell the position faster and more tax-efficiently than traditional direct indexing, their ability to hedge the risk of the position without selling it should be limited. Second, the optimized approach tends to overfit forecast risk reduction and may be ineffective at reducing realized risk, even when it offers a reduction in forecast risk. Both concerns are validated by the experiments summarized below.

Long-Short Optimized Approach and Forecast Risk Reduction

To give our optimized portfolios the best shot at hedging the concentrated stock, we construct them with the sole purpose of minimizing the overall portfolio TE—without considering any alpha model or transaction and financing cost penalties. Exhibit 6 shows the extent to which long-short optimized completion portfolios help reduce forecast TE. As seen in Panels A and B, greater leverage is more effective at reducing the forecast TE level when the concentrated stock position is large relative to the diversified part of the portfolio. The need for greater leverage diminishes as the concentrated stock position becomes smaller relative to the overall portfolio value.

Exhibit 6, Panel B, shows that, when the stock position is 80% and the diversified portfolio is 20% of the overall portfolio, compared to the stock-oblivious index, the 250/150 optimized portfolio reduces the mean and median forecast TE by 3.6% and 2.9%, respectively. With the same 80–20 weight allocation to the concentrated stock and the diversified portfolio, the long-only optimized portfolio reduces the mean and median forecast TE by only 0.7% and 0.5%, respectively.

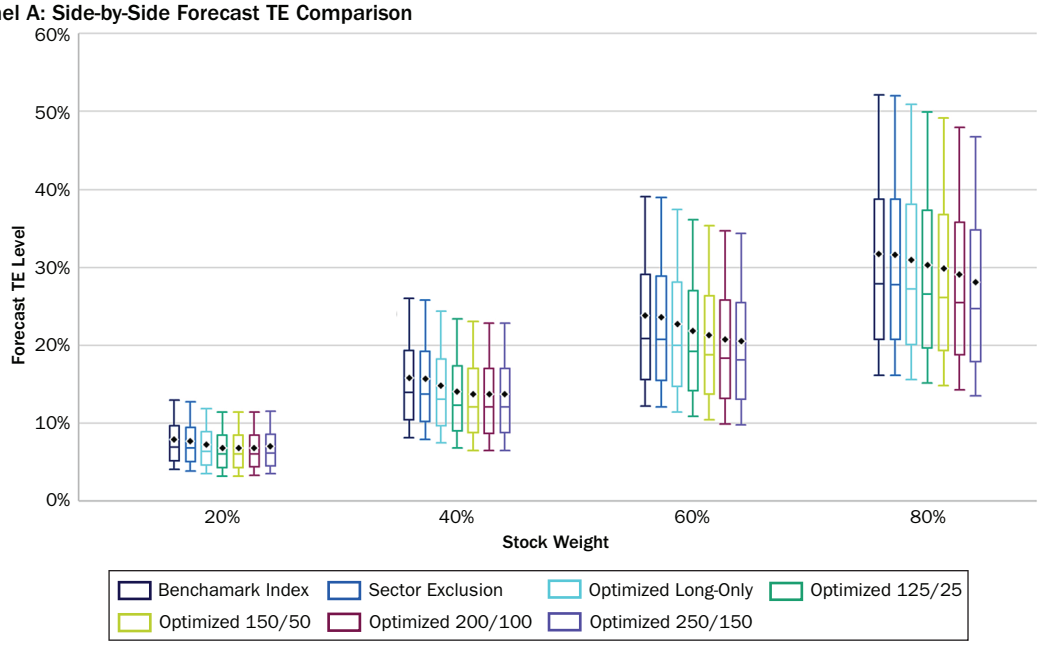
While the absolute risk reduction offered by the long-short optimized completion portfolios seems high, it is less impressive relative to the level of the forecast TE: when the stock weight is 80%, the mean and median ratios of forecast TE with long-short optimized completion portfolios to that with the stock-oblivious benchmark index are higher than 87%, meaning the risk reduction is no greater than 13% (see Exhibit 6, Panel C). This is significantly better than the long-only optimized portfolio (whose forecast TE is more than 97% of the stock-oblivious benchmark index on average—barely any difference), but in absolute terms it constitutes a very modest TE reduction, especially when the costs and complexity of long-short portfolios are considered.

As the stock position becomes smaller relative to the overall portfolio value, the importance of leverage in the completion portfolio declines. In fact, forcing too much leverage when the stock position is small actually may increase the forecast

EXHIBIT 6

Forecast TEs of All Types of Portfolios versus Benchmark Index Portfolio

Panel A: Side-by-Side Forecast TE Comparison



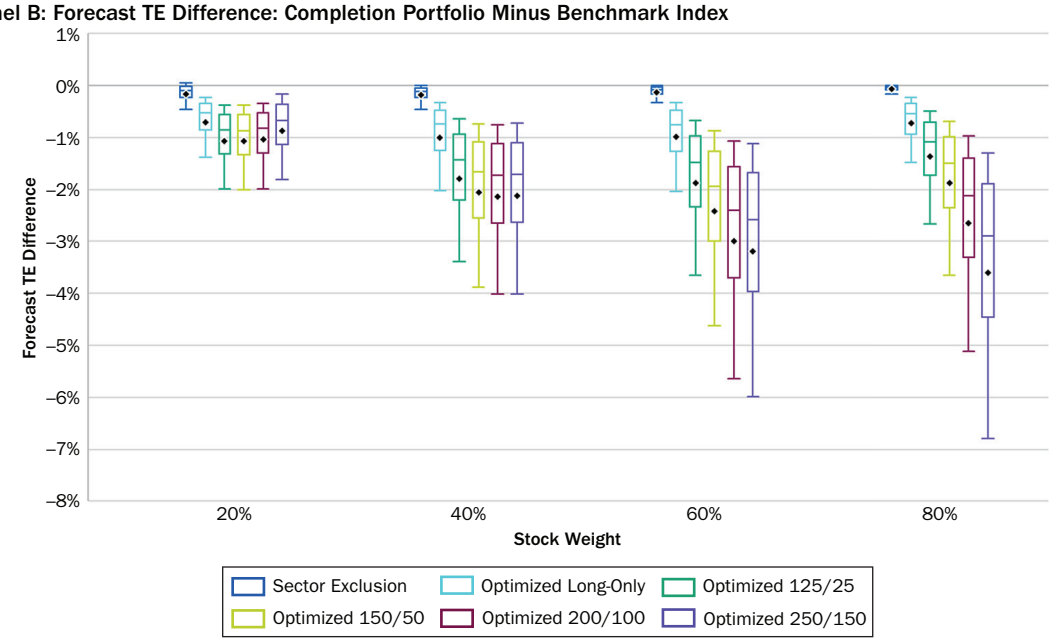
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	7.9%	7.0%	15.9%	13.9%	23.8%	20.9%	31.7%	27.8%
Sector Exclusion	7.8%	6.8%	15.7%	13.8%	23.6%	20.8%	31.6%	27.8%
Optimized Long-Only	7.2%	6.4%	14.9%	13.1%	22.8%	20.0%	31.0%	27.2%
Optimized 125/25	6.9%	6.0%	14.1%	12.4%	21.9%	19.3%	30.3%	26.6%
Optimized 150/50	6.9%	6.0%	13.8%	12.1%	21.4%	18.8%	29.8%	26.2%
Optimized 200/100	6.9%	6.1%	13.7%	12.1%	20.8%	18.3%	29.1%	25.5%
Optimized 250/150	7.0%	6.2%	13.7%	12.1%	20.6%	18.1%	28.1%	24.7%

(continued)

EXHIBIT 6 *(continued)*

Forecast TEs of All Types of Portfolios versus Benchmark Index Portfolio

Panel B: Forecast TE Difference: Completion Portfolio Minus Benchmark Index



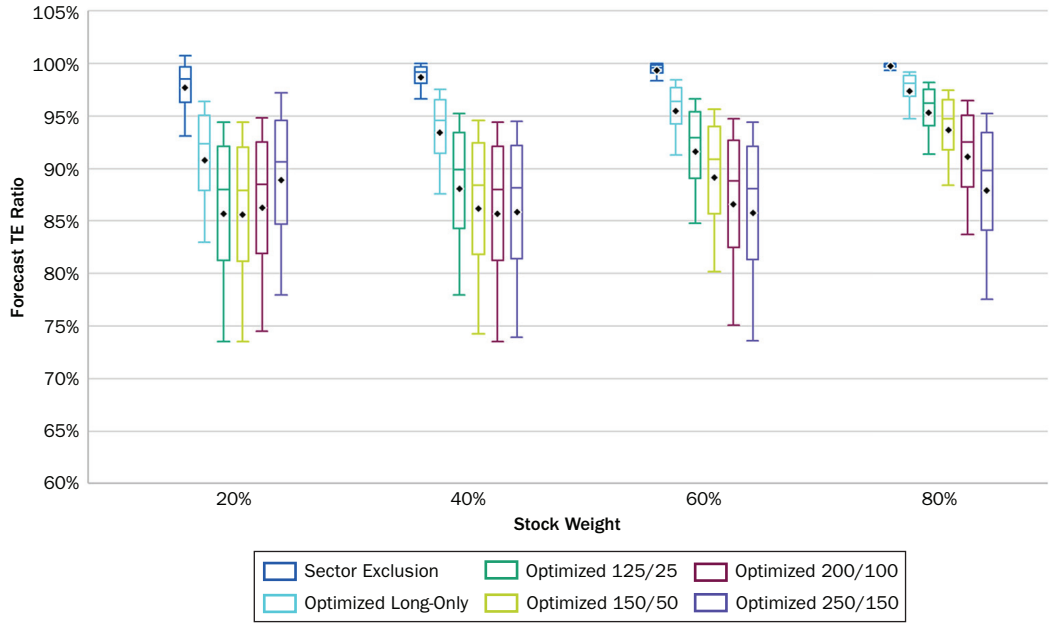
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	–0.2%	–0.1%	–0.2%	–0.1%	–0.1%	–0.1%	–0.1%	0.0%
Optimized Long-Only	–0.7%	–0.5%	–1.0%	–0.7%	–1.0%	–0.8%	–0.7%	–0.5%
Optimized 125/25	–1.1%	–0.9%	–1.8%	–1.4%	–1.9%	–1.5%	–1.4%	–1.1%
Optimized 150/50	–1.1%	–0.9%	–2.1%	–1.7%	–2.4%	–1.9%	–1.9%	–1.5%
Optimized 200/100	–1.0%	–0.8%	–2.1%	–1.7%	–3.0%	–2.4%	–2.7%	–2.1%
Optimized 250/150	–0.9%	–0.7%	–2.1%	–1.7%	–3.2%	–2.6%	–3.6%	–2.9%

(continued)

EXHIBIT 6 *(continued)*

Forecast TEs of All Types of Portfolios versus Benchmark Index Portfolio

Panel C: Forecast TE Ratio: Completion Portfolio Divided by Benchmark Index



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	97.7%	98.5%	98.7%	99.2%	99.4%	99.6%	99.7%	99.8%
Optimized Long-Only	90.8%	92.3%	93.4%	94.6%	95.5%	96.4%	97.4%	98.1%
Optimized 125/25	85.7%	88.0%	88.0%	89.9%	91.6%	93.0%	95.3%	96.2%
Optimized 150/50	85.6%	87.9%	86.1%	88.4%	89.2%	90.8%	93.6%	94.7%
Optimized 200/100	86.3%	88.5%	85.7%	88.0%	86.6%	88.8%	91.1%	92.5%
Optimized 250/150	88.9%	90.6%	85.9%	88.1%	85.7%	88.0%	87.9%	89.8%

NOTES: In this exhibit, the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

TE—see, for example, the 20% stock weight scenario and the 250/150 portfolio. Exhibit 6, Panel B, shows that the additional benefits of forecast TE reduction are minimal once leverage reaches 200/100 for the 60% stock weight, 150/50 for the 40% stock weight, and 125/25 for the 20% stock weight. The reduction in the level of forecast TE diminishes as the stock position becomes a smaller part of the overall portfolio. At the same time, the relative TE reduction, shown in Exhibit 6, Panel C, increases slightly.

Exhibit 7 shows which components of TE the long-short approach helps reduce. As seen in Exhibit 7, Panel A, the long-short approach reduces the common-factor contribution more effectively than the long-only approach, with leverage being particularly beneficial for TE reduction when the stock position has a large weight in the overall portfolio. On the other hand, as shown in Exhibit 7, Panel B, the stock-specific component remains virtually identical irrespective of the diversified portfolio type.

In sum, long-short portfolios are more effective than long-only at reducing common-factor risk. However, the main source of risk for concentrated stock—the stock-specific risk—remains unaddressed. As a result, while long-short portfolios reduce the forecast TE more than long-only, the magnitude of risk reduction remains very limited.

Long-Short Optimized Approach and Realized Risk Reduction

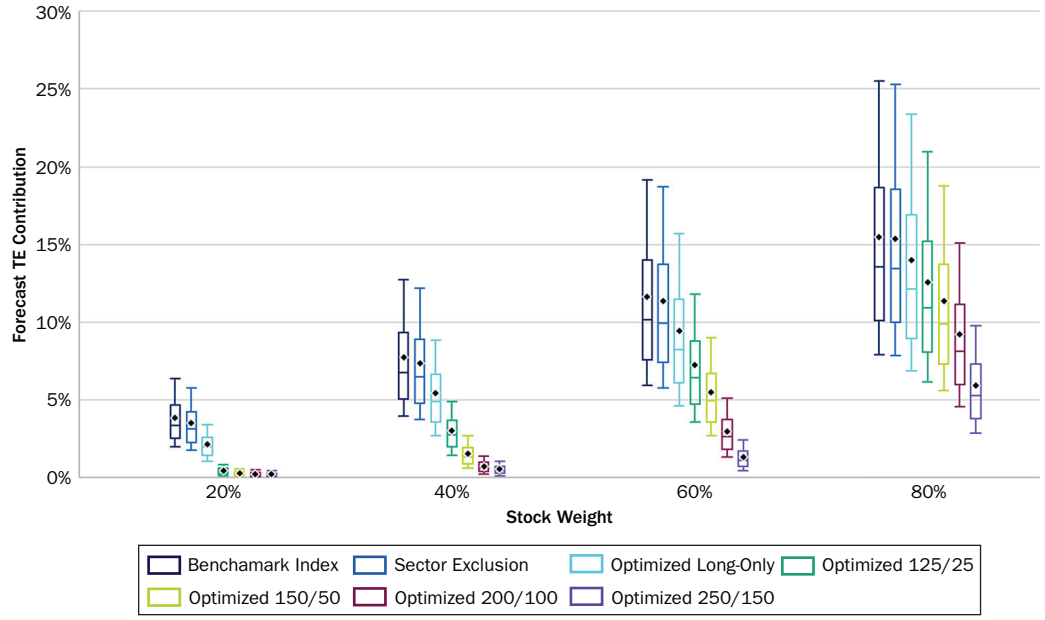
Is the reduction offered by long-short portfolios for forecast TEs preserved in realized TEs? We show the realized TEs for the long-short portfolios in Exhibit 8. Exhibit 8, Panel A, shows that optimized long-short portfolios produce very modest realized TE reductions compared to simply holding the stock-oblivious index portfolio. As seen in all prior experiments, the dominant source of TE reduction is simply having less concentrated stock in the overall portfolio.

Exhibit 8, Panel B, allows us to zoom in on these realized TE differences. The realized TE reduction is small relative to the level of TE. For example, the largest mean (median) realized TE reduction of 1.64% (1.28%) is observed for the 250/150 completion portfolio with the stock weight in the overall portfolio of 80%. In all cases, the realized TE reduction is substantially smaller than the forecast TE reduction observed in Exhibit 6, Panel B. Furthermore, a zero realized TE reduction is well within the 10th to 90th percentile range, indicating that it is not statistically significant. This is different from the forecast TE reduction seen in Exhibit 6, Panel B, where the 10th to 90th percentile ranges were substantially below zero for all the optimized completion portfolios and particularly for the long-short ones.

EXHIBIT 7

Forecast TE Decomposition for All Types of Completion Portfolios

Panel A: Common-Factor Forecast TE Contribution



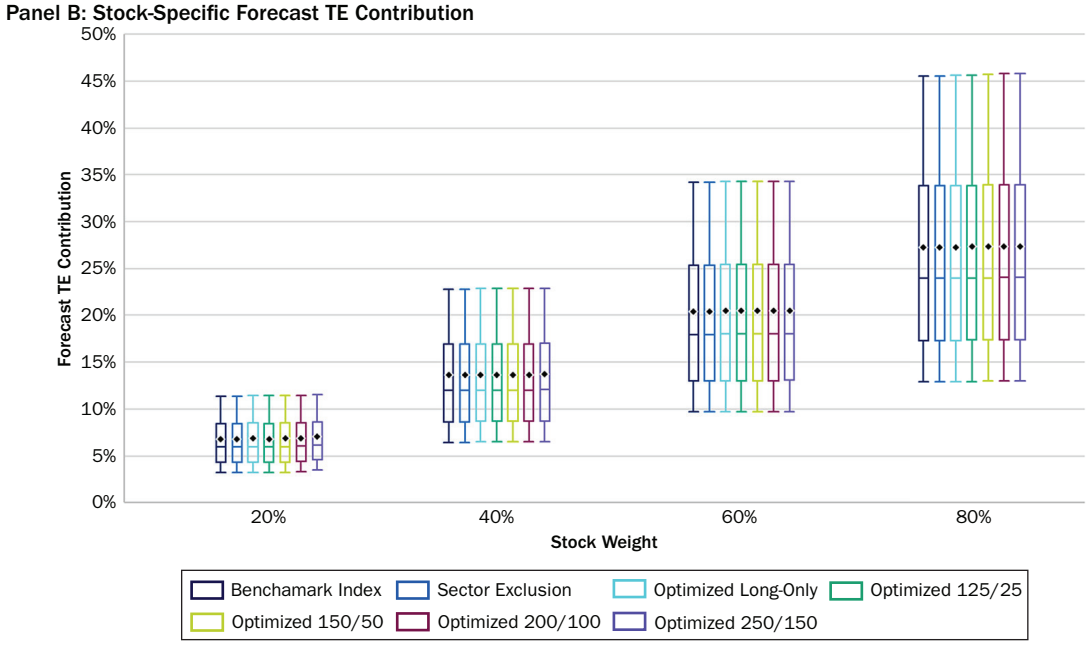
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	3.9%	3.4%	7.7%	6.8%	11.6%	10.2%	15.5%	13.5%
Sector Exclusion	3.5%	3.1%	7.4%	6.5%	11.4%	9.9%	15.4%	13.4%
Optimized Long-Only	2.1%	2.0%	5.5%	4.9%	9.5%	8.3%	14.0%	12.1%
Optimized 125/25	0.5%	0.4%	3.0%	2.8%	7.3%	6.4%	12.6%	10.9%
Optimized 150/50	0.3%	0.3%	1.5%	1.3%	5.5%	5.0%	11.3%	9.9%
Optimized 200/100	0.3%	0.2%	0.8%	0.6%	3.0%	2.6%	9.2%	8.1%
Optimized 250/150	0.2%	0.2%	0.6%	0.5%	1.3%	1.1%	6.0%	5.3%

(continued)

EXHIBIT 7 *(continued)*

Forecast TE Decomposition for All Types of Completion Portfolios

Panel B: Stock-Specific Forecast TE Contribution



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	6.8%	6.0%	13.6%	12.0%	20.4%	18.0%	27.3%	24.0%
Sector Exclusion	6.8%	6.0%	13.6%	12.0%	20.4%	18.0%	27.3%	24.0%
Optimized Long-Only	6.8%	6.0%	13.7%	12.0%	20.5%	18.0%	27.3%	24.0%
Optimized 125/25	6.8%	6.0%	13.7%	12.0%	20.5%	18.0%	27.3%	24.0%
Optimized 150/50	6.8%	6.0%	13.7%	12.0%	20.5%	18.0%	27.3%	24.0%
Optimized 200/100	6.9%	6.1%	13.7%	12.0%	20.5%	18.1%	27.4%	24.0%
Optimized 250/150	7.0%	6.2%	13.7%	12.1%	20.5%	18.1%	27.4%	24.1%

NOTES: In this exhibit, the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

Exhibit 8, Panel C, which shows the ratio of TEs with completion portfolios and those with the stock-oblivious index portfolio, leads to a similar conclusion. The realized TE ratio is substantially higher than the forecast TE ratio. Or, in other words, the realized percent TE reduction is substantially lower than the forecast TE reduction. Again, in contrast to the forecast TE ratios shown in Exhibit 6, Panel C, the ratio of one is now strictly within the 10th to 90th percentile range, indicating that the reduction in realized TE is not statistically significant.

FROM INSIGHTS TO SOLUTIONS

Let us summarize what we have learned thus far. First, for a single stock, volatility is driven primarily by the stock-specific component. Common-factor risk related to industry and style-factor exposures plays a very limited role in the total single-stock volatility.

Second, excluding the stock's sector is largely ineffective at reducing both the forecast and the realized risk that the stock contributes to a portfolio.

Third, the optimized completion portfolio approach reduces the forecast common-factor risk arising from the concentrated stock. Adding leverage and shorting further reduces the common-factor risk, particularly when the stock position is a large fraction of the overall portfolio. While completion portfolios reduce the common-factor risk, they leave the dominant source of risk—the stock-specific risk—unaddressed. In all scenarios, however, the forecast risk reduction is quite limited. Furthermore, the risk reduction benefits of optimized completion portfolios observed in forecast TE largely disappear when realized TEs are considered. This is true for both the long-only and the long-short optimized completion portfolios.

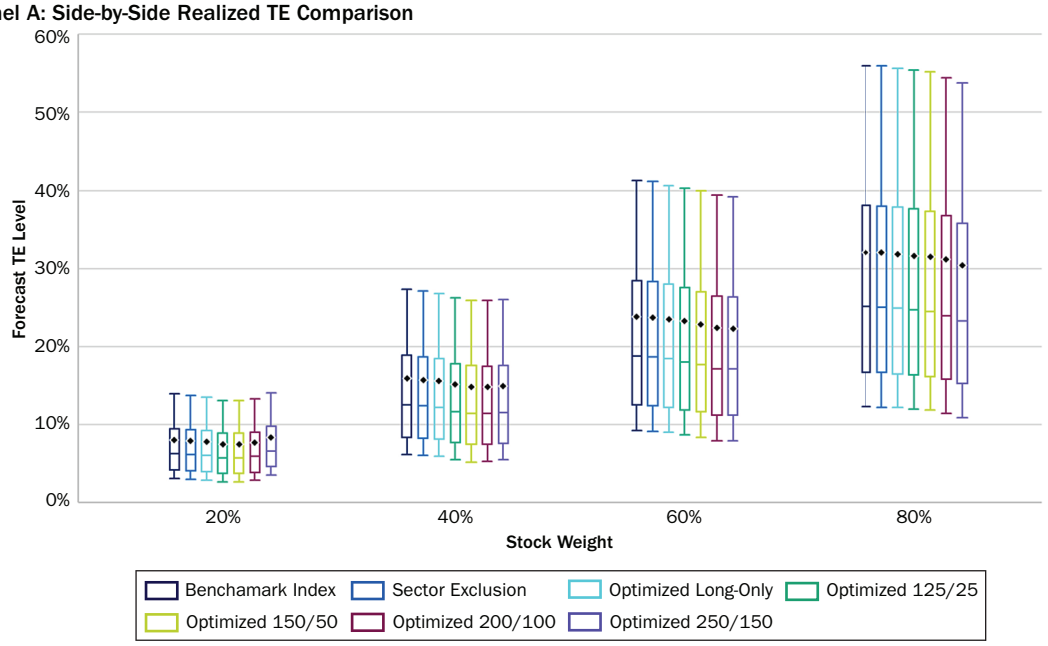
Fourth, the most prominent result in all the experiments is a significantly lower TE of the portfolios that hold a smaller concentrated stock position. This result is equally strong for forecast and realized TEs.

As a result of these observations, we believe that the investor's primary focus should be on selling the concentrated position. The benefit of diversifying the portfolio by reducing its concentration is unambiguous, while the benefit of hedging the stock's exposures via a completion portfolio approach is questionable, particularly when it comes to realized risk.

EXHIBIT 8

Realized TEs of All Completion Portfolios versus Benchmark Index Portfolio

Panel A: Side-by-Side Realized TE Comparison



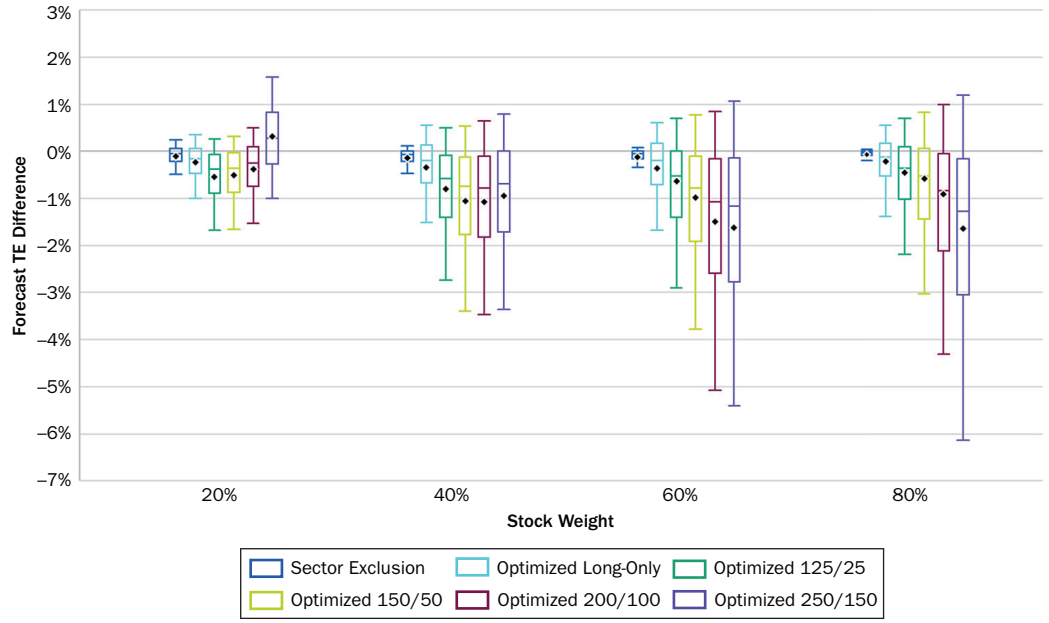
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Benchmark Index	8.1%	6.3%	15.9%	12.5%	23.9%	18.8%	32.1%	25.1%
Sector Exclusion	8.0%	6.2%	15.8%	12.4%	23.8%	18.7%	32.0%	25.1%
Optimized Long-Only	7.8%	6.0%	15.6%	12.2%	23.5%	18.4%	31.9%	24.9%
Optimized 125/25	7.5%	5.8%	15.1%	11.7%	23.2%	18.1%	31.6%	24.7%
Optimized 150/50	7.5%	5.8%	14.9%	11.4%	22.9%	17.7%	31.5%	24.5%
Optimized 200/100	7.7%	5.9%	14.8%	11.4%	22.4%	17.2%	31.2%	24.0%
Optimized 250/150	8.4%	6.6%	15.0%	11.6%	22.3%	17.1%	30.4%	23.3%

(continued)

EXHIBIT 8 *(continued)*

Realized TEs of All Completion Portfolios versus Benchmark Index Portfolio

Panel B: Realized TE Difference: Completion Portfolios Minus Benchmark Index



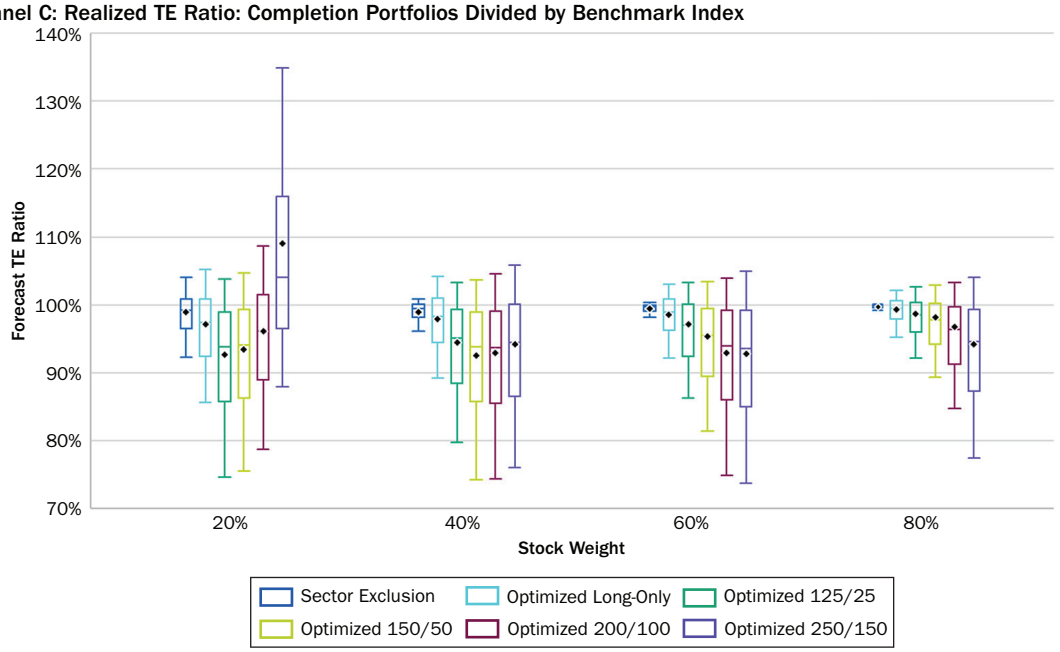
	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	–0.10%	–0.06%	–0.15%	–0.07%	–0.12%	–0.06%	–0.07%	–0.03%
Optimized Long-Only	–0.23%	–0.16%	–0.34%	–0.20%	–0.37%	–0.20%	–0.22%	–0.13%
Optimized 125/25	–0.55%	–0.38%	–0.80%	–0.58%	–0.64%	–0.53%	–0.45%	–0.35%
Optimized 150/50	–0.52%	–0.37%	–1.06%	–0.74%	–0.98%	–0.78%	–0.58%	–0.53%
Optimized 200/100	–0.37%	–0.25%	–1.08%	–0.77%	–1.49%	–1.07%	–0.92%	–0.84%
Optimized 250/150	0.31%	0.27%	–0.95%	–0.68%	–1.62%	–1.16%	–1.64%	–1.28%

(continued)

EXHIBIT 8 *(continued)*

Realized TEs of All Completion Portfolios versus Benchmark Index Portfolio

Panel C: Realized TE Ratio: Completion Portfolios Divided by Benchmark Index



	Stock Weight							
	20%		40%		60%		80%	
	Mean	Median	Mean	Median	Mean	Median	Mean	Median
Sector Exclusion	98.9%	99.2%	99.0%	99.4%	99.5%	99.7%	99.8%	99.9%
Optimized Long-Only	97.2%	97.4%	98.0%	98.3%	98.6%	98.9%	99.3%	99.5%
Optimized 125/25	92.7%	93.8%	94.5%	95.1%	97.1%	97.0%	98.6%	98.5%
Optimized 150/50	93.5%	94.1%	92.6%	93.9%	95.4%	95.6%	98.2%	97.7%
Optimized 200/100	96.2%	96.1%	92.9%	93.7%	93.0%	94.0%	96.8%	96.4%
Optimized 250/150	109.0%	104.1%	94.2%	94.5%	92.8%	93.5%	94.2%	94.6%

NOTES: In this exhibit, the box denotes the 25th, 50th, and 75th percentiles. The whiskers denote the 10th and 90th percentiles. The marker represents the mean.

This makes tax-aware strategies that combine strong pre-tax return potential with reliable capital loss realization particularly beneficial. On the one hand, the investor may enjoy pre-tax performance, possibly in excess of that of the benchmark index, and on the other, may utilize the losses realized by the strategy to offset the gains realized upon selling the concentrated stock.

These results by no means contradict the practical reality that completion portfolios make a conceptually prudent choice by accounting for concentrated stock positions. Rather, our findings allow concentrated stock investors and their advisors to understand the relative benefit of hedging the stock (with unrelated stocks) compared to selling the stock. This understanding can facilitate the prioritization and evaluation of the various steps in the concentrated stock diversification process, where selling the stock quickly and tax-efficiently should be the highest priority.

With this in mind, in the next section, we use a set of four tax-aware strategies to evaluate the speed at which the concentrated stock position can be liquidated in a tax-efficient manner. The strategies were originally modeled in Liberman et al. (2023) and do not seek to hedge the concentrated stock. As in Liberman et al. (2023), the long-short strategies are alpha-seeking strategies, managed with an eye toward tax efficiency.

THE CONCENTRATED POSITION LIQUIDATION HORSE RACE

Tax-Aware Strategies and Concentrated Stock Liquidation Process

We sample concentrated stock returns from a lognormal distribution. The market is assumed to have an arithmetic mean return of 8% and a volatility of 15%. The stock is assumed to have a zero alpha and a beta of 1.0—resulting in the stock's arithmetic mean return of 8%—and a volatility of 40%. We assume that the stock pays no dividends, such that the value of the stock is simply its cumulative return. Each stock is assumed to have a zero cost basis, meaning that the entire value of the stock is an unrealized gain.

We sample tax-aware strategy losses for a specific month since inception from strategy simulations replicating Liberman et al. (2023). We then pair randomly drawn concentrated stock values (that is, cumulative randomly drawn returns) with randomly drawn losses for one of the four tax-aware strategies and for a specific stock-strategy weight allocation. For example, at the inception of the liquidation process, if the stock weight is assumed to be 80% of the overall portfolio, the allocation to the tax-aware strategy is 20% of the portfolio, while if the stock weight is 20% of the overall portfolio, the allocation to the tax-aware strategy is 80%. For each scenario, we make 10,000 random draws and show the median results.

At the end of each month, we observe the loss realized by a given strategy for that month and sell a portion of the concentrated stock position, ensuring that the gain recognized upon selling the stock exactly matches the loss realized by the strategy. We reinvest the proceeds from the sale into the strategy. If no loss is realized by the tax-aware strategy, no sale of the stock occurs. In this process, the capital loss from the strategy entirely offsets the capital gain from the stock, leaving the investor with a net capital result (gain or loss) of zero.

In other words, we keep alternating between two steps—realizing the loss and selling shares of the stock (reinvesting the sale proceeds into the strategy)—until the concentrated position is fully liquidated and all the capital is invested in the strategy. Again, no loss from the strategy means no stock liquidation. Making the strategy completely independent of the concentrated stock allows us to avoid modeling such complexities as financing long and short extensions using as collateral a portfolio that contains a large concentrated position (more on this point below), or hedging the concentrated stock with diversified portfolio positions, thereby impacting the ability of the long-short portfolios to implement the alpha model.[11]

As previously mentioned, we consider four tax-aware strategies. The long-only direct-indexing strategy maximizes capital losses subject to a constant 1% TE target within its own segment of the overall portfolio. The tax-aware 150/50, 200/100, and 250/150 strategies seek to maximize pre-tax alpha subject to constant TE targets of 2%, 4%, and 6%, respectively, while at the same time realizing capital losses. All the strategies target a beta of 1 with respect to the Russell 1000 Index. Importantly, the strategies do not specifically seek to accelerate losses to speed up the liquidation of the stock. This is particularly relevant for the three tax-aware long-short strategies, whose main objective is to deliver pre-tax alpha in a tax-efficient manner.

Concentrated Position Liquidation Results

Let us first consider the case where the concentrated stock is a small fraction of the overall portfolio—20%, while the diversified portfolio is 80%. In other words,

[11] In reality, to prevent wash sales and straddles, we exclude the concentrated stock from the tax-aware strategy. However, given the high diversification of the strategy portfolios, this hads no measurable impact on our results. Therefore, for the sake of simplicity, we do not model the exclusion of the concentrated stock from the strategy portfolios.

at the start of the process, the diversified portfolio is four times larger than the concentrated position. This scenario is shown in Exhibit 9, Panel A. In the median outcome, the long-short strategies help to fully liquidate the concentrated stock in just a few months. With the direct-indexing strategy, full liquidation of the stock takes longer but is still accomplished in under two years. In other words, when the stock position is a small fraction of the overall portfolio, all types of tax-aware strategies allow the investor to liquidate it reasonably quickly.

What if the investor does not have the luxury of a diversified portfolio valued at a multiple of the concentrated stock? Exhibit 9, Panels B and C, show scenarios in

EXHIBIT 9

Liquidation of Concentrated Stock under Alternative Scenarios

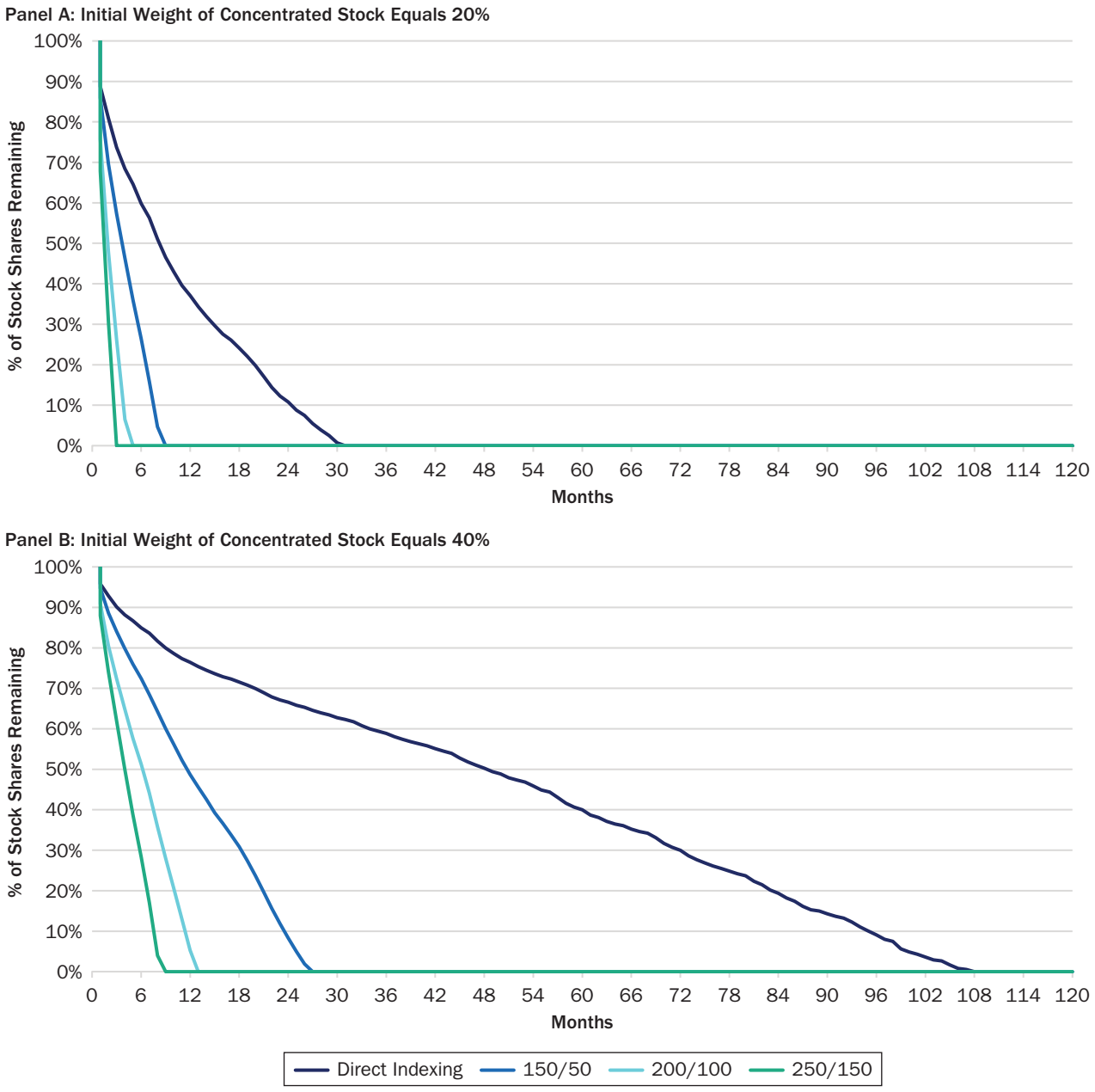


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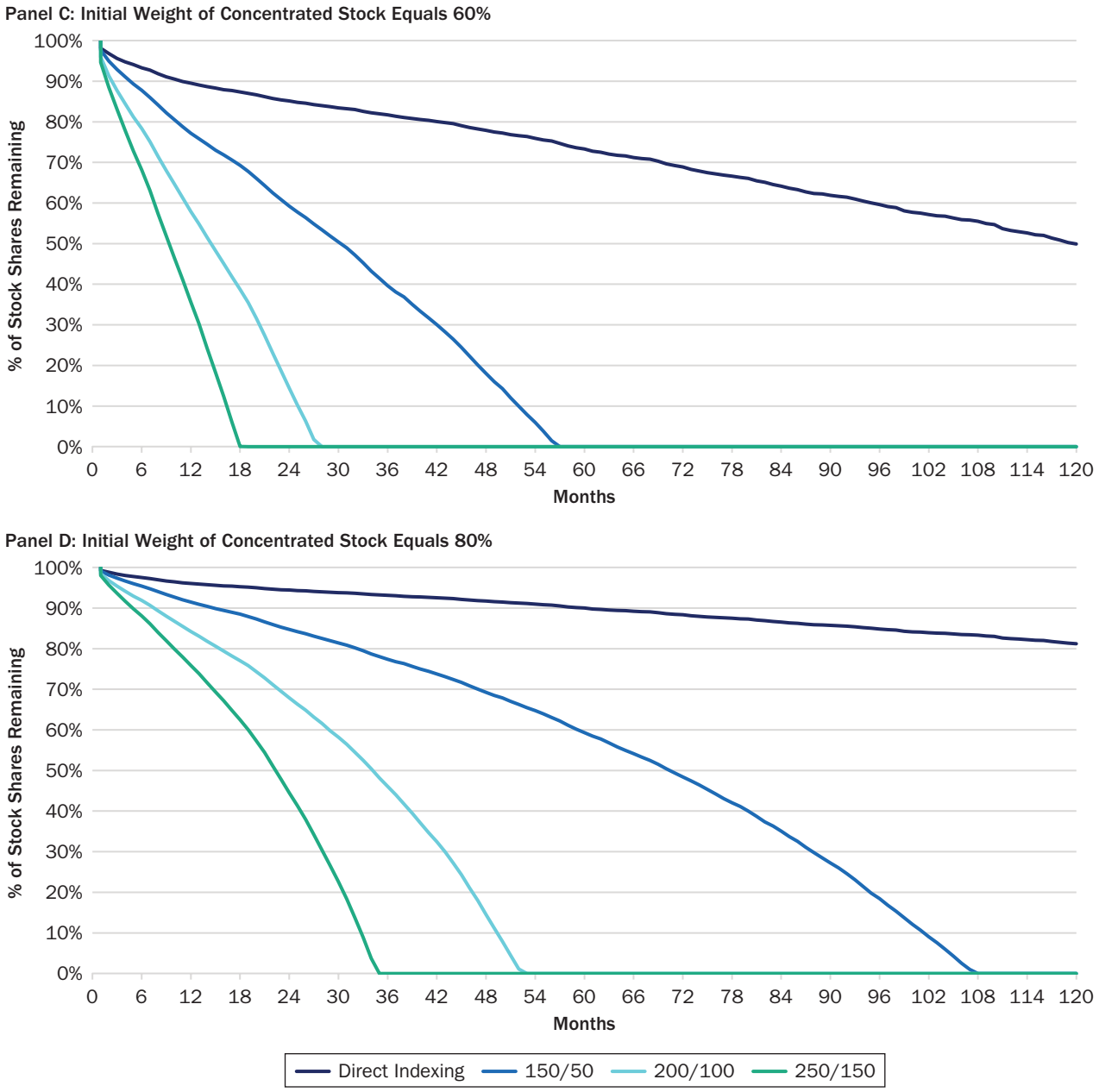
EXHIBIT 9 *(continued)*

Liquidation of Concentrated Stock under Alternative Scenarios



which the diversified portfolio and the concentrated stock position have similar values at the start of the diversification process. In Panel B, the stock position is a little less than half of the overall portfolio, at 40%, while in Panel C, the position is a little more than half, at 60%. In these two cases, the higher leverage strategies—250/150 and 200/100—allow the investor to fully liquidate the stock within a 1–3-year period. The 150/50 strategy extends the liquidation period to 2–4.5 years. Finally, with the direct-indexing strategy, it takes from 7 to more than 10 years to fully liquidate the stock. In sum, the long-short strategies, particularly the higher leverage and TE ones, still offer reasonable stock liquidation periods, while direct indexing struggles with speedy liquidation.

Now let us consider a scenario in which the investor has limited funds outside of the concentrated position—for example, with 80% of the portfolio in the concentrated stock. In this case, the direct-indexing strategy is of little help. Exhibit 9, Panel D, shows that with this strategy, even after 10 years, the investor still holds more than 80% of the initially held shares of the stock. In contrast, the 250/150 and 200/100 strategies allow the investor to fully liquidate the position over periods of four and five years, respectively. With the 150/50 strategy, it now takes nearly a decade to fully liquidate the stock.

Understanding the Overall Portfolio Risk

An argument can be made that, although it takes significantly longer to liquidate a concentrated stock position with direct indexing, at least the direct-indexing strategy has a lower TE than its long-short counterparts. A rebuttal to that argument is that TE should be measured at the overall portfolio level, not just for the tax-aware strategy in isolation. This is exactly what we do in this section.

Exhibit 10 shows the overall portfolio forecast TE corresponding to the concentrated stock liquidation schedules shown in Exhibit 9. For larger initial stock weights, such as 60% and 80% (see Exhibit 10, Panels C and D), the overall portfolio forecast TE with direct indexing remains higher than that with long-short strategies. Specifically, for the 80% initial stock weight shown in Panel D, after 10 years, the TE with direct indexing is more than three times higher than that with any of the long-short strategies.

In addition, the TE of the long-short strategies is intentional and is undertaken with the explicit purpose of beating the benchmark index and generating excess pre-tax returns, while the TE arising from a concentrated stock is unintended and is not expected to be compensated by excess returns. Furthermore, as shown in Liberman et al. (2023), the long-short strategy TE can be modulated up or down in a tax-efficient manner to fit the investor's preferences and risk profile.[12]

Recap of Concentrated Position Liquidation Horse Race

We summarize the concentrated position liquidation results in Exhibit 11. Panel A shows the number of months until full liquidation. The length of time to full liquidation decreases with the leverage of the tax-aware strategy and increases with the initial weight of the concentrated stock. Notably, the 200/100 and 250/150 strategies allow the investor to fully liquidate the stock within a reasonably short timeframe—under five years and three years, respectively, even when the allocation to the strategy is only 20%.

Exhibit 11, Panel B, shows the forecast TE of the overall portfolio at the start of the liquidation process. Long-short strategies in the diversified part of the portfolio lead to a slightly higher initial TE of the overall portfolio, especially when the weight of the diversified portfolio is large relative to the weight of the concentrated stock.[13] In contrast, when the weight of the concentrated stock is large, the impact of the forecast TE of the diversified strategy on the TE of the overall portfolio is negligible—see, for example, the 80% concentrated stock weight scenario.

Full liquidation of the concentrated stock means that the overall portfolio TE reaches its target. For convenience, in Exhibit 11, Panel C, we show where the

[12] One caveat is that if the strategy's TE and leverage are reduced before the stock diversification process is complete, it may slow the process.

[13] As a reminder, tax-aware long-short strategies deviate from benchmark weights for the purpose of generating excess pre-tax returns: the higher the tracking error, the greater the return generation potential.

risk-reduction process is going—specifically, we provide the forecast TE targets for the investor's portfolio after the concentrated stock is fully liquidated.

The TE of portfolios holding concentrated stock is high. For example, when the stock is 80% of the portfolio, the portfolio's forecast TE is about 30%. Every tax-aware long-short strategy allows the investor to fully liquidate the concentrated stock and achieve the target TE level within the 120-month period that we model here. The only strategy that fails to do so is direct indexing in the scenarios with an initial stock weight of 60% and 80% (see Exhibit 11, Panel A). When the initial stock weight is 60%,

EXHIBIT 10

Forecast TE of the Overall Portfolio during Concentrated Stock's Liquidation

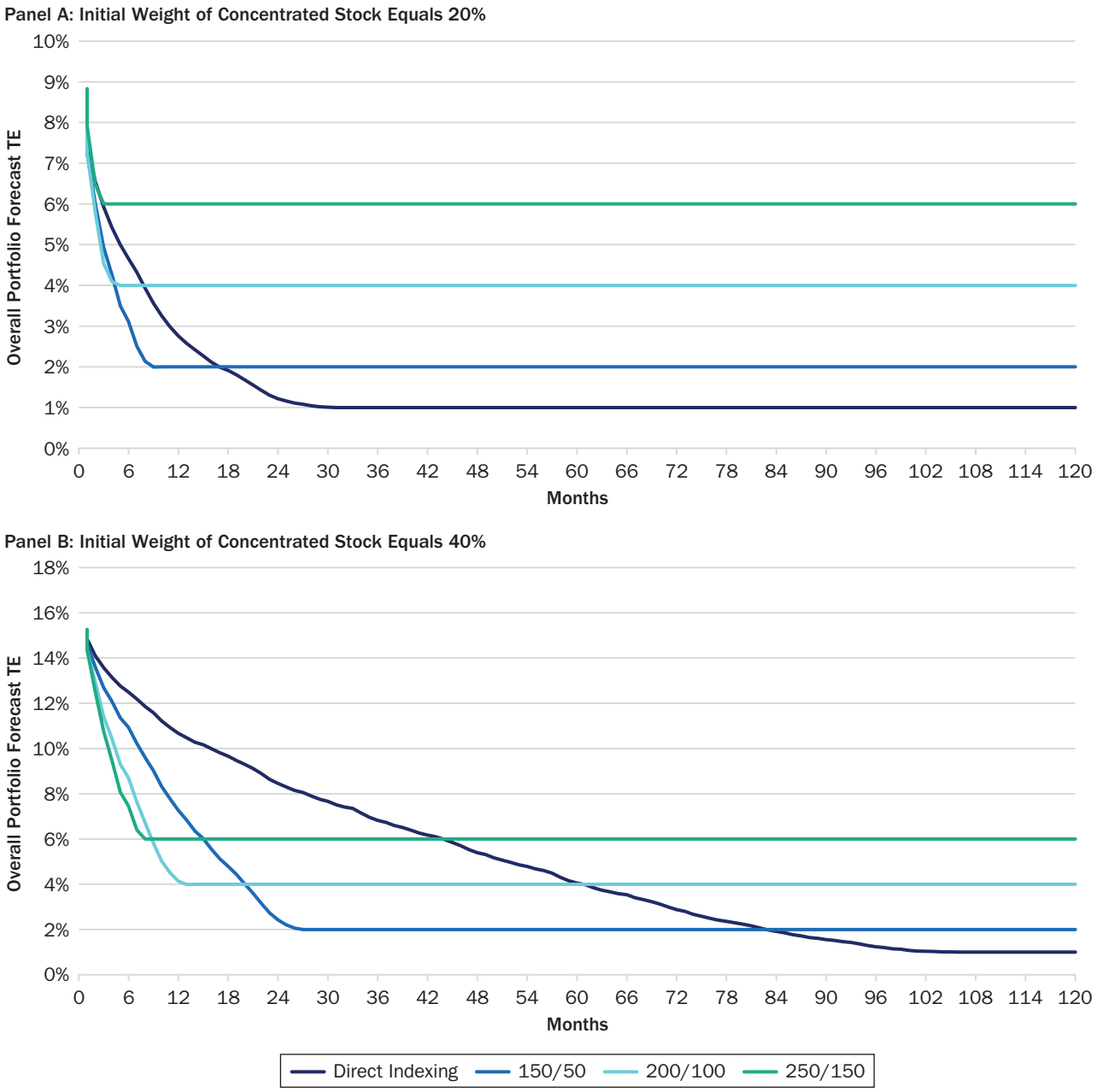


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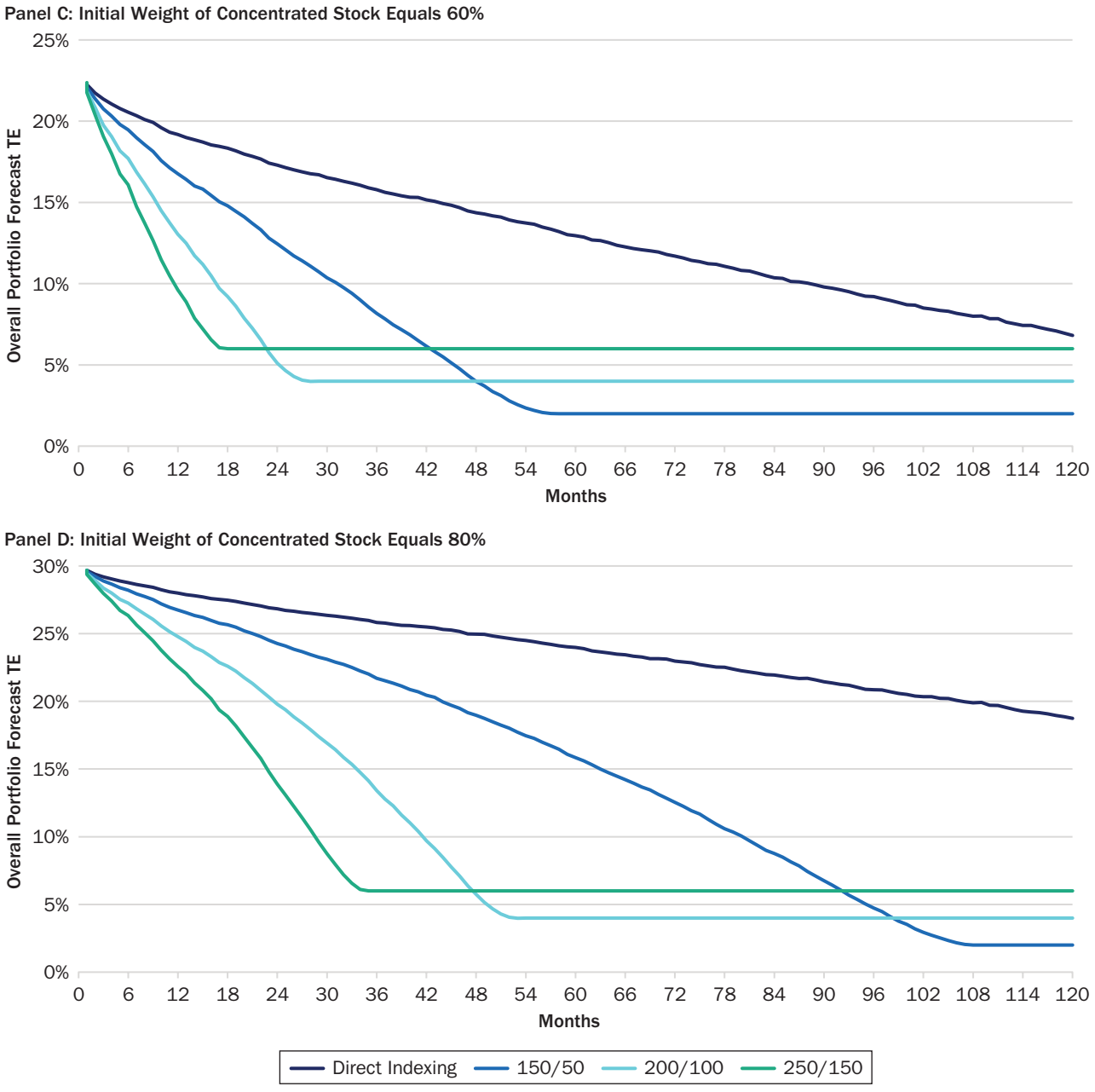
EXHIBIT 10 *(continued)*

Forecast TE of the Overall Portfolio during Concentrated Stock's Liquidation



at the end of the 120-month period, direct indexing leaves the investor with 50% of the initial shares of the stock and a forecast TE of 6.8% (compared to the starting TE of 22.3%). When the initial stock weight is 80%, at the end of the 120-month period, the investor retains 81% of the initial shares of the stock and holds a portfolio with a forecast TE of 18.7% (compared to the starting TE of 29.7%).

Limitations of the Analysis and Ideas for Further Research

The purpose of our simple experiment was to show the relative effectiveness of different tax-aware strategies. Optimizing after-tax wealth is outside the scope of this

EXHIBIT 11

Summary of Concentrated Position Liquidation Horse Race Results

Initial Weight of Concentrated Stock	Direct Indexing	150/50	200/100	250/150
Panel A: Months to Full Liquidation				
20%	31	9	5	3
40%	108	27	13	9
60%	>120	57	28	19
80%	>120	108	53	35
Panel B: Initial Forecast TE				
20%	7.5%	7.6%	8.1%	8.8%
40%	14.8%	14.9%	15.0%	15.3%
60%	22.3%	22.3%	22.3%	22.4%
80%	29.7%	29.7%	29.7%	29.7%
Panel C: Post-Liquidation Forecast TE Target				
	1%	2%	4%	6%

article. As a result, we would like to highlight three important constraints that could be relaxed in future research.

First, we assumed that the investor has no tolerance for realizing capital gains. If the investor were willing to realize some gains to reduce the concentration risk, the sale of the concentrated position could occur significantly faster. Although some taxes would be paid along the way, the reduction in volatility could compensate the investor for the taxes incurred while diversifying the concentrated stock exposure. This is particularly important for the direct-indexing strategy, which, while not being able to fully offset the gains from selling the stock, still could provide value during the diversification process.

Second, we assumed that the investor continues to hold the higher leverage strategies over the entire 10-year investment horizon. However, as demonstrated in Liberman et al. (2023), the strategy's TE and leverage can be reduced in a highly tax-efficient manner if the investor so desires. For example, when the stock position is 80% of the overall portfolio (see Exhibit 9, Panel D), it takes about three years to fully liquidate the stock with the 250/150 strategy and about four and a half years with the 200/100 strategy. After that, the investor can start gradually reducing the TE and leverage of the strategy. As shown in Liberman et al. (2023), while the reduction from 6% and 4% TE to 2%, and from 250/150 and 200/100 leverage to 150/50, can take several years, the optimized reduction is nonlinear, with most of it occurring within a one-year period.

Finally, we assumed that the investor cannot use any of the concentrated stock as collateral for creating leverage in the long-short strategies. While financing terms on concentrated positions may be inferior to those on diversified portfolios, concentrated positions still have borrowing power, especially when they become a smaller part of the overall portfolio.[14] The additional leverage increases the potential for both pre-tax alpha generation and capital loss realization.

[14] This concept was previously explored in Goldberg, Cai, and Selwitz (2022), in the context of concentrated portfolios containing several securities, and in Sosner and Krasner (2021), in the context of diversified portfolios used to fund a long-short strategy. These studies can be extended to model the de-risking of a single concentrated, low-basis stock.

EXHIBIT 12

Alternative Structures of the Long-Short Strategy Portfolio

	Concentrated Stock Used as Collateral	
	No	**Yes**
Account Collateral		
Concentrated Stock	$40	$40
Other Capital	$60	$60
Utilized as Collateral	**$60**	**$100**
Account Borrowing		
Magin Loan (150% times Collateral)	$90	$150
Short Borrow (150% times Collateral)	$90	$150
Portfolio Composition		
Concentrated Stock	$40	$40
Diversified Long Positions	$150	$210
Diversified Short Positions	$90	$150
Net Asset Value (All Longs minus Shorts)	$100	$100
Gross Notional Value (All Longs plus Shorts)	$280	$400

Exhibit 12 helps illustrate this point. Consider, for example, the 40% concentrated stock scenario and the 250/150 strategy. The first column shows the assumptions used in the analysis shown in Exhibits 9 and 10 above. We assumed that 150% extensions only applied to the diversified 60% part of the portfolio. That is, for $100 invested in the overall portfolio, the collateral utilized to finance long and short extensions was $60. This resulted in a margin loan of $90 and borrowed short positions of $90. That is, the portfolio consisted of a $40 concentrated position, a $150 diversified long portfolio ($60 of the original capital plus $90 financed with a margin loan), and a $90 diversified short portfolio.

If, on the other hand, the concentrated position also were used as collateral for leverage, as shown in the second column of Exhibit 12, the 150% extensions would be applied to $100, resulting in a $210 long diversified portfolio (= $60 initial capital plus $150 of margin loan used to purchase long positions) and a $150 short portfolio. Given that the long and short extensions are now 67% larger—$150 versus $90—the speed of selling the concentrated stock can also be significantly increased.

PRACTICAL IMPLICATIONS

Before concluding, we would like to summarize the practical implications of our article. First, completing a concentrated stock position using a portfolio of unrelated stocks fails to reduce the most significant source of risk associated with a single stock—its stock-specific risk. Furthermore, substantial additional capital may be required to fund a portfolio intended to complete the concentrated stock. However, as our results show, greater additional capital makes the overall portfolio less risky not because the risk of the concentrated stock is hedged more effectively, but simply because the stock represents a smaller fraction of the portfolio. Advisors still can utilize the completion portfolio approach if it reduces emotional stress on the investor and sets the investor on the path to diversification. However, advisors should be aware—and also inform their investors—of the ineffectiveness of the completion portfolio approach for hedging the risk of a concentrated stock.

Second, the most certain way to reduce concentration risk is to sell a concentrated position. Tax-aware strategies can help the investor improve the tax efficiency of the concentrated stock diversification process. Given the ineffectiveness of the completion portfolio approach for hedging the risk of a concentrated stock, advisors should de-emphasize the supposed hedging aspect of completion portfolios.[15] Instead, they should focus on those features of the diversified part of the overall portfolio that have the most practical relevance for the investor: the capacity to realize capital losses, which can help offset realized capital gains from selling a concentrated stock; the ability to maintain the target TE of the diversified part of the portfolio; and, for tax-aware long-short strategies, the potential to deliver high pre-tax alpha. Importantly, tax-aware long-short strategies, which have the propensity for realizing greater capital losses than direct indexing, could be a preferred choice when a concentrated stock position represents a large part of the overall portfolio and needs to be reduced quickly and tax-efficiently. Furthermore, the higher the leverage of a long-short strategy, the faster the stock can be liquidated.[16]

CONCLUSION

We show that, for a typical concentrated stock, the majority of the risk comes from the stock-specific component. This risk cannot be hedged with other unrelated stocks; the only way to reduce it is by selling the concentrated position.

Sector exclusions have little effect on hedging either the forecast or realized risk of concentrated stock. Optimized long-only completion portfolios help hedge the forecast common-factor risk of the stock better than sector exclusions. However, since the common-factor risk component is only a small fraction of the total concentrated stock risk, the risk reduction provided by optimized portfolios is very limited. Optimized long-short completion portfolios are more effective at hedging this small common-factor component of *forecast* concentrated stock risk than their long-only counterparts, and their effectiveness increases with leverage. Unfortunately, the evidence for the ability of optimized portfolios—long-only and long-short portfolios alike—to hedge *realized* concentrated stock risk is mixed at best.

Where does this leave us? We believe that sector exclusion and optimized completion approaches still may add value. A priori, they seem like prudent portfolio management. If anything, they may encourage investors and their advisors to engage in diversifying concentrated stock. That said, the main conclusion of our article is that investors and their advisors should have realistic expectations about the practical effectiveness of completion portfolios when it comes to de-risking concentrated stock. Creating a completion portfolio can be the first—and the least effective—step in the process, the main purpose of which should be to sell the concentrated stock.

We show that implementing a tax-aware strategy in the diversified part of the overall portfolio is crucial for selling the concentrated stock in a tax-efficient manner. Traditional direct-indexing strategies may help when the concentrated stock is a relatively small part of the overall portfolio. Long-short strategies are far more effective than direct indexing at facilitating tax-efficient concentrated position reduction. Another significant advantage they have over direct indexing is their potential for realizing pre-tax alpha, which helps the investor enhance long-term wealth accumulation.

[15] The only exception is that, when a tax-aware long-short strategy is used, the strategy cannot short the concentrated stock. Shorting the concentrated stock would be treated as a constructive sale of the stock.

[16] This is in addition to the higher pre-tax alpha potential, which is discussed in detail in Liberman et al. (2023).

APPENDIX

RISK OF CONCENTRATED STOCKS AND PORTFOLIOS

Separating Market Risk from Other Sources of Common-Factor Risk

In this article, we use the Barra USE3L risk model as the risk model for portfolio construction.[17] The USE3L model does not have a dedicated market risk factor; instead, market risk is captured as part of industry risk. In some of our experiments, we need to separate the market risk contribution from the industry risk contribution. Here is how we perform the separation.

Let $\boldsymbol{w}$ and $\boldsymbol{b}$ be $N \times 1$ vectors of portfolio and benchmark index weights, respectively, where N is the number of stocks. The benchmark index represents a broadly diversified market portfolio. A concentrated stock portfolio will be represented by $\boldsymbol{w}^T = [0, \ldots, 1, \ldots, 0]$, where the stock has a weight of 1 and every other stock has a weight of 0. A portfolio that allocates 80% of the weight to concentrated stock i and 20% of the weight to a diversified benchmark index will be represented by $\boldsymbol{w}^T = [0.2 \times b_1, \ldots, 0.2 \times b_{i-1}, 0.8 + 0.2 \times b_i, 0.2 \times b_{i+1}, \ldots, 0.2 \times b_N]$. Let Σ be a $N \times N$ covariance matrix. We define the market-orthogonal portfolio $\boldsymbol{z}$ as

$$\boldsymbol{z} = \boldsymbol{w} - \beta \boldsymbol{b} \tag{A-1}$$

where $\beta = \dfrac{\boldsymbol{w}^T \Sigma \boldsymbol{b}}{\boldsymbol{b}^T \Sigma \boldsymbol{b}}$.

Using the definition in Equation A-1, it is easy to show that the forecast covariance of the market-orthogonal portfolio with the benchmark index is zero:

$$Cov(\boldsymbol{z}, \boldsymbol{b}) = \boldsymbol{z}^T \Sigma \boldsymbol{b} = 0 \tag{A-2}$$

Equation A-1 can be rewritten as

$$\boldsymbol{w} = \beta \boldsymbol{b} + \boldsymbol{z} \tag{A-3}$$

Following the decomposition of portfolio weights in Equation A-3, the portfolio's variance can be decomposed as

$$Var(\boldsymbol{w}) = \beta^2 Var(\boldsymbol{b}) + Var(\boldsymbol{z}) + 2 \times Cov(\boldsymbol{z}, \boldsymbol{b}) \tag{A-4}$$

Note, however, that, according to Equation A-2, the last term in Equation A-4 is 0. Therefore, Equation A-4 reduces to

$$Var(\boldsymbol{w}) = \beta^2 Var(\boldsymbol{b}) + Var(\boldsymbol{z}) \tag{A-5}$$

We consider the first term in Equation A-5, $\beta^2 Var(\boldsymbol{b})$, to be the market risk component and decompose the second term $Var(\boldsymbol{z})$ using a standard risk decomposition into common-factor and stock-specific components. The common-factor component is then further decomposed into industry and style-factor components.[18]

[17] We always lag the risk model's covariance matrix by one month. Lagging the covariance matrix by a month ensures that the covariance matrix used for volatility forecasts is available at the time of portfolio formation, as it takes a few days after month-end for the risk model to be released. This is particularly important for comparisons of forecast and realized risk of optimized completion portfolios.

[18] Style factors in the Barra USE3L risk model include Value, Size, Momentum, Volatility, etc.

Single-Stock Volatility Decomposition

Using the methodology described above, we decompose single-stock forecast volatility into four components: market risk, industry risk, style-factor risk, and stock-specific risk. The stocks are sampled from the Russell 3000 Index constituent universe as described in the Methodology section in the main text. Exhibit A1, Panel A, shows the mean stock forecast volatility decomposition by sector and volatility quintile. In Exhibit A1, Panel B, we rescale the risk contributions to sum to 100%.

The exhibit allows us to make several important observations. First, the forecast volatility of a mean stock is quite high, although it varies by sector. For example, in the Energy, Health Care, Information Technology, and Telecom sectors, more than half the stocks have a forecast volatility above 50%. Second, the stock-specific component dominates a typical single stock's risk: well above 50% of the total volatility is stock-specific. Third, the common-factor risk is dominated by market risk, while industry and style-factor risk are typically a small component of both the common-factor and the total risk. These observations also hold true across volatility quintiles.

Portfolio Tracking Error Decomposition

Whereas the previous section focused on total stock volatility, here we focus on the stock's tracking error (TE) to a benchmark index. We apply the same decomposition approach as above to the TE of portfolios composed of the concentrated stock and a benchmark index portfolio—for example, 80% concentrated stock and 20% index portfolio. The decomposition starts with defining portfolio weights as

$$\boldsymbol{w} = \theta \boldsymbol{s} + (1-\theta)\boldsymbol{b} \tag{A-6}$$

where $\boldsymbol{s}^T = [0, \ldots, 1, \ldots, 0]$ is the vector with the value of 1 corresponding to the concentrated stock and the value of 0 corresponding to every other stock, $\boldsymbol{b}$ is the vector of benchmark index weights, and θ is the weight of the stock in the portfolio. Note that $\sum_{i=1}^{N} w_i = 1$.

We compute the vector of active portfolio weights by subtracting the vector of benchmark weights from the vector of portfolio weights:

$$\boldsymbol{a} = \boldsymbol{w} - \boldsymbol{b} \tag{A-7}$$

Note that, since $\sum_{i=1}^{N} w_i = 1$ and $\sum_{i=1}^{N} b_i = 1$, $\sum_{i=1}^{N} a_i = 0$. Substituting Equation A-6 into A-7, we obtain

$$\boldsymbol{a} = \theta \boldsymbol{s} + (1-\theta)\boldsymbol{b} - \boldsymbol{b} = \theta(\boldsymbol{s} - \boldsymbol{b}) \tag{A-8}$$

Effectively, the active portfolio is long θ of the concentrated stock and short θ of the benchmark index portfolio. The greater the weight of the concentrated stock, θ, the larger the active weights.

Using Equation A-8, we can describe the TE of the portfolio $\boldsymbol{w}$ consisting of θ fraction of the concentrated stock and $1 - \theta$ fraction of the benchmark index portfolio as

$$\mathrm{TE}(\boldsymbol{w}) = \sqrt{\boldsymbol{a}^T \Sigma \boldsymbol{a}} = \theta \sqrt{(\boldsymbol{s}-\boldsymbol{b})^T \Sigma (\boldsymbol{s}-\boldsymbol{b})} \tag{A-9}$$

In other words, the TE of portfolio $\boldsymbol{w}$ is proportional to the TE of portfolio $\boldsymbol{s}$:

$$\mathrm{TE}(\boldsymbol{w}) = \theta \times \mathrm{TE}(\boldsymbol{s}) \tag{A-10}$$

where

$$\mathrm{TE}(\boldsymbol{s}) = \sqrt{(\boldsymbol{s}-\boldsymbol{b})^T \Sigma (\boldsymbol{s}-\boldsymbol{b})} \tag{A-11}$$

As a result, for the purposes of TE decomposition, we only show the decomposition of Equation A-11, which corresponds to the TE of a portfolio consisting of a single stock. The TE of any other portfolio and its decomposition scales proportionately with the weight of the concentrated stock, θ.

We continue to use the same sample of stocks as in Exhibit A1: the stocks are sampled from the Russell 3000 Index constituent universe as described in the Methodology section in the main text. The benchmark portfolio is the Russell 3000 Index portfolio.

EXHIBIT A1

Single-Stock Forecast Volatility Decomposition

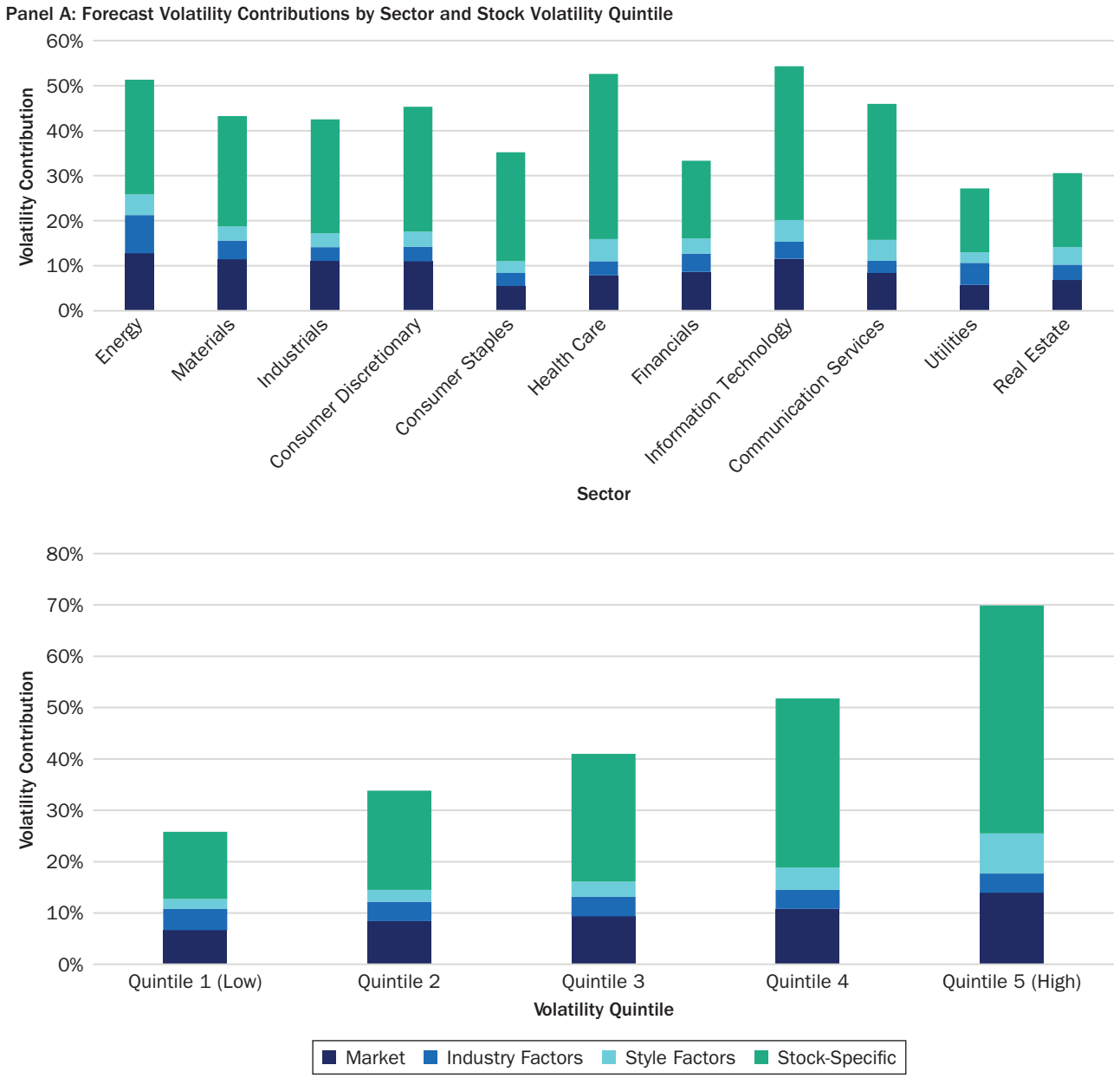


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EXHIBIT A1 *(continued)*

Single-Stock Forecast Volatility Decomposition

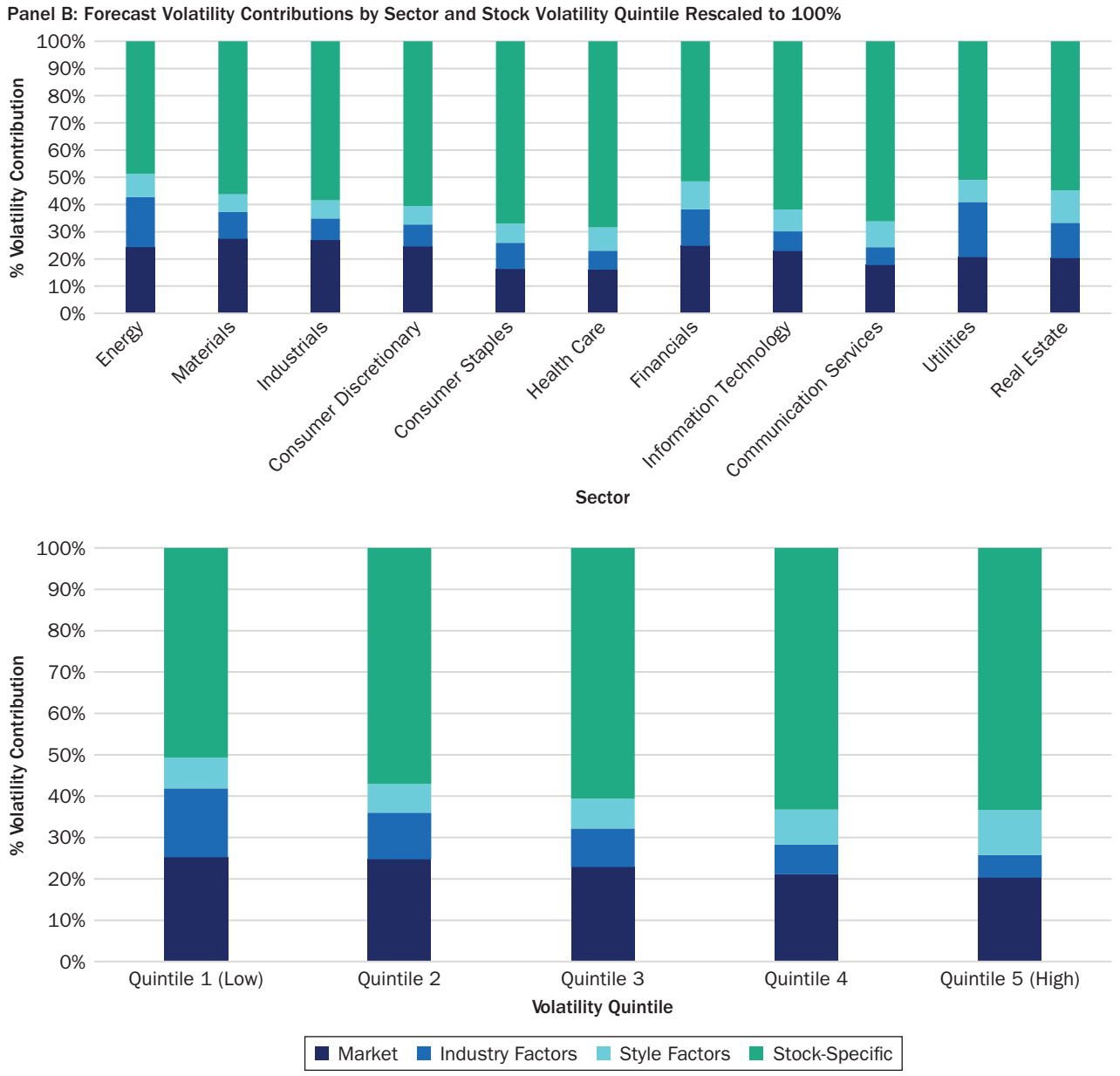
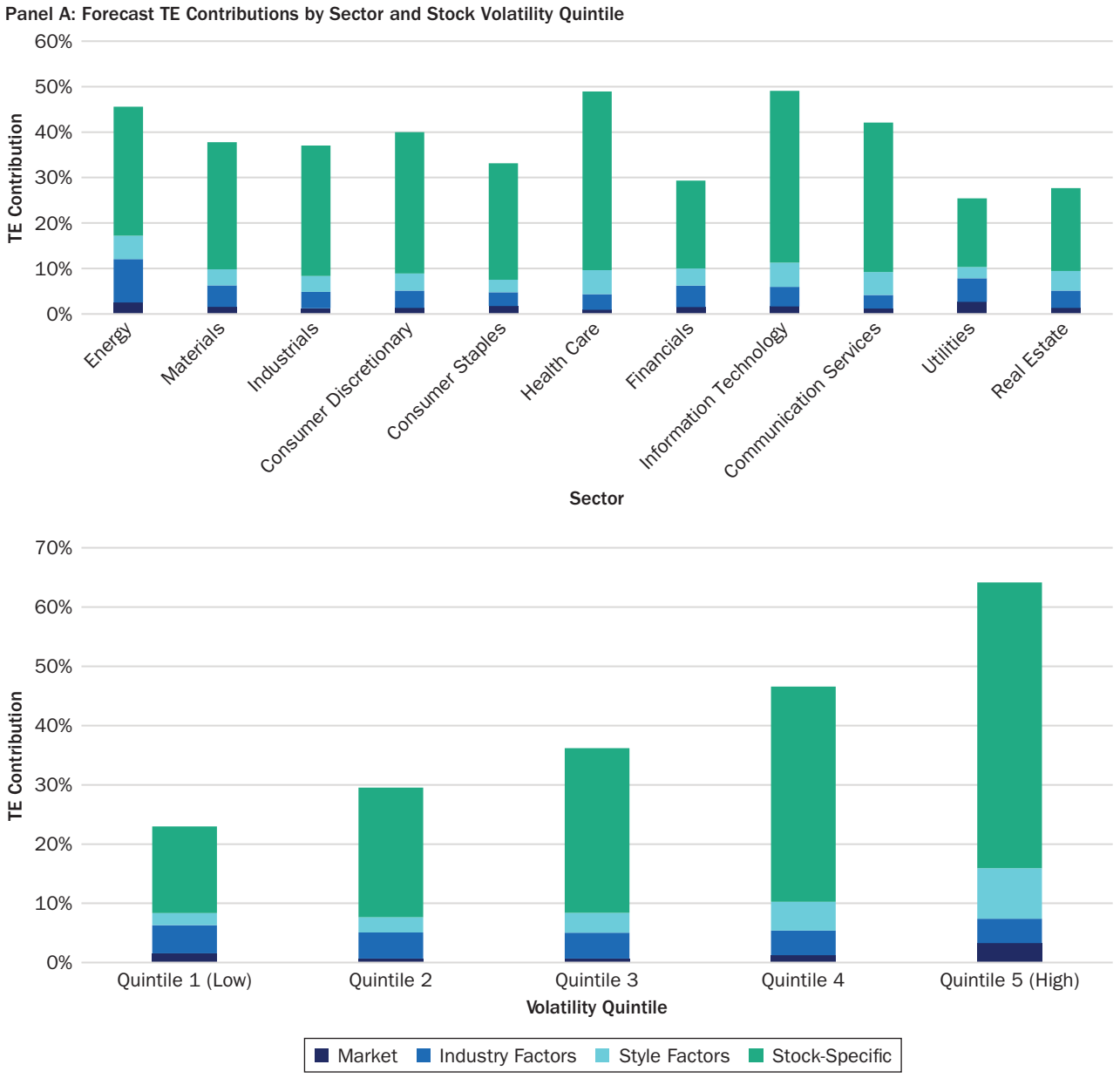


Exhibit A2, Panel A, shows single-stock forecast TE decomposition levels, while Panel B shows the decomposition rescaled to 100%. The TE of the portfolios is dominated by stock-specific risk. This is true for stocks in all sectors and in all volatility quintiles.

As we show in Equation A-10, the TE values shown in Exhibit A2, Panel A, can be used to compute the TE for different concentrated stock weights in the portfolio. For example, for the average stock in the top volatility quintile, the TE to the Russell 3000 Index is 64.2%. When such a stock has a 20% weight in the portfolio, the TE is 12.8%, while when the stock's weight is 80%, the TE is 51.3%.

EXHIBIT A2

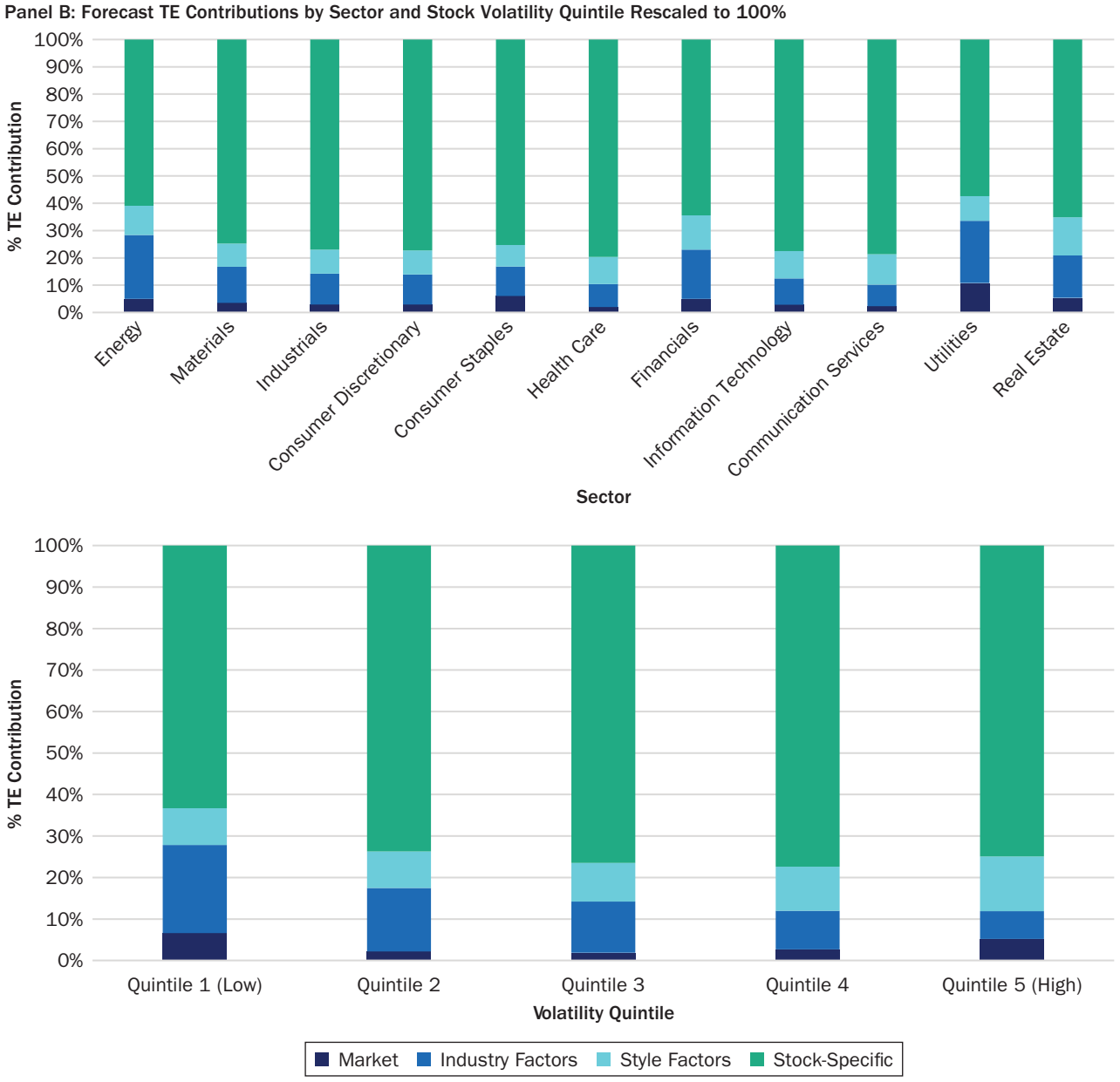
Single-Stock Forecast TE Decomposition



(continued)

EXHIBIT A2 *(continued)*

Single-Stock Forecast TE Decomposition



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